Preference Learning in School Choice Problems*

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May 20, 2026

Abstract

We study a school choice model in which students acquire costly information about their idiosyncratic preferences before submitting rank-order lists to a mechanism. We show that the well-known inefficiency of the DA mechanism, stemming from its strategy-proofness, is further compounded by disincentivized preference learning. A self-reinforcing feedback loop emerges: strategy-proofness ensures students can apply to ex-ante superior schools without risk, which crowds these schools and reduces acceptance probabilities, thereby weakening the incentive to learn idiosyncratic preferences and further homogenizing rank-order reports. We allow for flexible information acquisition in which students choose both the quantity and focus of information gathered.

1 Introduction

The standard model of school choice literature since Abdulkadiroğlu and Sönmez (2003) is to adopt Gale and Shapley (1962)'s two-sided matching model where students have preferences over potential matches and schools have priorities over students, accounting for factors such as test scores and proximity.

This paper offers a new perspective on a centralized school choice model in which students incur a cost to gather and process information about their idiosyncratic preferences. In our view, what is often referred to as *preferences* in the standard model are better understood as *expectations* of idiosyncratic preferences that reflect the perceived suitability of different schools. Families do not have firsthand experience with different schools, as they do not regularly consume or purchase them. To learn their idiosyncratic

*We thank Yuichi Imai for excellent research assistance and Mariagiovanna Baccara, Aaron Bodoh-Creed, Francesco Fabbri, Anqi Li, Doron Ravid, Leeat Yariv, and seminar participants for valuable comments.
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preferences, families gather information from various sources, such as school district websites, parent forums, and review and rating websites such as GreatSchools.org, niche.com, or SchoolDigger.com. The aggregation of available information — e.g., reading brochures, attending open houses, and browsing online forums — into application decisions consumes considerable time and energy, so students hold at best interim expectations of their preferences.

We compare the efficiency loss across the two most prominent school choice mechanisms: the Immediate-Acceptance (also known as the "Boston") and the Deferred-Acceptance (DA) mechanisms. The DA mechanism has been widely favored for its strategy-proofness and fairness — often referred to as no justified envy" or "stability".[1] However, the literature has also documented efficiency losses under DA, particularly when students have similar preferences (Miralles (2009), Abdulkadiroğlu et al. (2011), Pathak and Sönmez (2008), Featherstone and Niederle (2016)).

If students were fully informed about their preferences, the potential efficiency loss of the DA mechanism relative to the Boston mechanism is straightforward to understand. Both mechanisms assign students to schools in a series of rounds, where in each round students apply to their top remaining choice. If a student is rejected, they proceed to their next choice in the following round. The Boston mechanism treats matches made in each round as permanent, so students who apply to competitive schools risk losing the opportunity to match with lower-ranked schools if rejected. The DA mechanism, on the other hand, defers all matches until the final round. Students rejected by a higher-ranked school can move down their rank-order list and apply to lower-ranked schools, potentially displacing previously tentative matches. This property incentivizes truthful preference reporting, making DA strategy-proof. This strategy-proofness is, however, the source of inefficiency: students who are nearly indifferent between schools may apply to a more competitive school first, forcing out students who have a strong preference for that school.

The above comparison of mechanisms may not reflect the complete picture. In reality, students hold at best interim expectations of their preferences, and any given mechanism induces a game in which students must make decisions about information acquisition. Students and their families choose how much information to gather about their preferences based on the admission odds of each school. They may opt not to seek information about a school, or gather information only about exceptional fit, if the admission probability is low. Consequently, learning decisions are shaped by the mechanism in use and the learning and reporting strategies of other students. It

[1] Both terms mean that any school a student prefers over their assigned match is allocated only to students with higher priority for that school.

is therefore inaccurate to evaluate the inefficiency of each mechanism taking interim expected preferences as if they were true preferences.

This paper identifies an additional source of efficiency loss under the DA mechanism: reduced information acquisition, beyond what the literature has already documented. We build a minimal model that can demonstrate the potential efficiency loss of the DA mechanism. The model consists of a unit mass of infinitesimal students and three schools: s (superior in expectation), a (average), and b (below average). Match payoffs are $u_a = 1$, $u_b = 0$, and $u_s = v + \theta$, respectively. While $v > 1/2$ is common across students, θ is an idiosyncratic preference shock distributed uniformly over $[0, 1]$, independently across students. For illustration, assume equal capacities of $1/3$ for all three schools.

A given mechanism — Boston or DA — defines a Bayesian game in which each student first acquires information about her unobservable preference shock and then submits a rank-order list. We adopt the Rational Inattention (RI) framework for information acquisition, first introduced by Sims (1998) and Sims (2003). The RI framework allows for flexible information acquisition, enabling students to choose not only the quantity of information they gather but also its focus. The value of different forms of information varies with the mechanism in use, and students must prioritize the information most critical to their application decisions in order to minimize information costs.

As a complete information benchmark, assume students observe their preference shocks. Since $\min\{u_a, u_s\} > u_b$ with probability 1, students submit either sab or asb under either mechanism. A majority of students report sab rather than asb, as a majority of them prefer school s over a ($\Pr[v + \theta > 1] = v > 1/2$). Since the DA mechanism is strategy-proof, a student reports sab if and only if $v + \theta > 1$, assuming no indifference ($v + \theta \neq 1$). Reporting sab under the Boston mechanism, on the other hand, is risky: a failure to match in the first round is likely to trigger another rejection in the second round, resulting in a match with school b. Thus, a student reports sab only if her preference type θ is significantly greater than $1 - v$ (Lemma 2). The DA mechanism, by contrast, does not discourage nearly indifferent students from reporting sab, leading to more homogeneous rank-order submissions. Consequently, the mechanism relies more heavily on random tie-breaking in assigning students to schools, resulting in a larger number of nearly indifferent students being matched with the in-demand school s. As a result, the allocation produced by the DA mechanism is less efficient than that of the Boston mechanism (Corollary 1).

Next, suppose information acquisition becomes costly. A student's rank-order submission is only partially informed by her unobservable preference type. Under either mechanism, a student is more likely to receive a signal recommending *sab* when her preference type is higher. The optimal signal structure and the accuracy of cost-justifying signals depend on the mechanism and the strategies of other students (Lemma 3). We characterize the equilibrium learning and reporting strategies under both the Boston and DA mechanisms (Proposition 1). Our equilibrium analysis shows that the DA mechanism continues to exhibit efficiency loss relative to the Boston mechanism (Proposition 2 and Corollary 2), and this loss becomes increasingly pronounced as the marginal cost of information rises (Proposition 6).

At first glance, it might seem straightforward that students acquire more information under the Boston mechanism, since applying to a competitive school first entails the risk of multiple rejections. This intuition, however, is incomplete: this same risk may also discourage information acquisition altogether, as students may opt not to apply to a highly competitive school in the first place.

A more precise intuition for the results is as follows. First, consider the Boston mechanism. Due to the risk associated with applying to school s, a student whose unobservable preference type is near $1 - v$ would submit *asb* regardless of whether her type falls above or below $1 - v$. There is little value in learning whether her preference type is only slightly above or below $1 - v$. Optimal information acquisition under the Boston mechanism therefore focuses on learning whether the preference type exceeds a cutoff substantially higher than $1 - v$ (Lemma 2).

Now suppose the mechanism switches to DA, while all students' information and reporting strategies are held fixed from the Boston mechanism. A student now prefers to submit *sab* if and only if her unobserved preference type θ exceeds $1-v$. Her optimal information acquisition strategy thus focuses around the cutoff $1 - v$, learning whether her preference type falls above or below that value. If her type exceeds $1 - v$, she is more likely to report *sab*; if it falls below $1 - v$, she is more likely to report *asb*. Since the relevant cutoff shifts down from the Boston benchmark to $1 - v$, more students end up submitting *sab*. The increased homogeneity of rank-order reports leads to greater reliance on random tie-breaking, which in turn discourages costly information acquisition. Consequently, each student's rank-order report is, in expectation, more likely to be *sab*, reflecting ex-ante expected utilities ($E[u_s] = v + \frac{1}{2} > u_a = 1$). This creates a self-reinforcing cycle between homogeneous rank-order reports and reduced information acquisition, further exacerbating the efficiency loss under the DA mechanism.

Lastly, it is worth noting that the DA mechanism is not always less efficient than the Boston mechanism, as shown in more complex school choice environments by (Troyan, 2012; Calsamiglia and Miralles, 2015).[2] While a more comprehensive school choice environment is desirable, our aim is to construct a simple setup that clearly demonstrates the logic behind the potential efficiency loss of the DA mechanism.

1.1 Related Literature

Chen and He (2021) also investigates information acquisition in school choice and compares welfare outcomes across mechanisms. Their model assumes that students first choose to learn their ordinal preferences and then their cardinal utilities, each at a cost, and finds that students need not learn cardinal utilities under DA since the mechanism is strategy-proof. In contrast, we adopt a flexible information acquisition framework, which allows for comparative statics and accommodates the possibility that more information may be acquired under DA depending on equilibrium acceptance probabilities.

Other studies have also explored the role of preference learning in matching. Immorlica et al. (2020) examines the design of a stable outcome by considering a process in which each student sequentially selects a school to investigate, and defines stability in terms of matching outcomes and students' beliefs about preferences. Bade (2015) considers heterogeneous learning costs among students and finds that prioritizing agents with higher information costs through a serial dictatorship approach can lead to greater information acquisition and enhanced efficiency. Harless and Manjunath (2018) examines students' choices of which school to learn about, while Kloosterman and Troyan (2020) investigates how students learn about others' preferences.

We study a mechanism design problem with rationally inattentive (RI) agents, emphasizing that a mechanism choice must account for the information agents choose to acquire. Other papers in this vein include Yang (2020) and Li and Yang (2020) on contract design with an RI agent, Bloedel and Segal (2018) on information design with an RI agent, Ravid (2020) on ultimatum bargaining between a seller and an RI buyer, and Cusumano et al. (2024) on firms competing in price against an RI consumer.

We measure information acquisition costs using mutual information. Shannon (1948), Cover and Thomas (2012), and Caplin and Dean (2015) provide information-theoretic, coding-theoretic, and revealed-preference foundations for this modeling choice, respectively. The growing literature on RI is surveyed by Caplin (2016) and Maćkowiak et al. (2023).

[2]If different top schools prioritize students differently — for instance, based on neighborhood criteria — the fear of being matched with an inferior school under the Boston mechanism may push students towards safer options, such as their neighborhood schools, rather than exploring potentially welfare-enhancing assignments.

2 Model

A unit mass of students must be assigned to three schools: s, which is superior in expectation; a, which is average; and b, which is below average. Each school j has a capacity $\lambda_j > 0$ such that $\sum_{j\in\{s,a,b\}} \lambda_j = 1$. Student preferences are represented by cardinal utilities, with $u_s = v + \theta$, $u_a = 1$, and $u_b = 0$. The constant $v \in (0, 1)$ is common to all students, while θ is an idiosyncratic preference shock, independently and uniformly distributed on $[0, 1]$ across students. The realization of θ determines each student's preference between schools s and a: a student with $v + \theta > 1$ obtains a higher match payoff from school s than from school a. The value v is also the fraction of students with such ex-post preferences. Regardless of the realization of θ, school b is always the least preferred option.

A matching mechanism requests each student to submit a rank-order list of either sab or asb, with school b always ranked last as the least preferred option. The mechanism then assigns students to schools based on the populations of students reporting sab and asb, subject to capacity constraints. We consider two mechanisms: the Immediate-Acceptance (Boston) mechanism and the Deferred-Acceptance (DA) mechanism, both of which are described in detail below.

Before participating in a given mechanism, students can acquire information about their unobservable preference shock θ at a cost. The signal structure $\Pi : [0, 1] \to \Delta\mathcal{Z}$ captures the information acquired by a student, where $\Pi(\cdot \mid \theta)$, $\forall\theta \in [0, 1]$, specifies a probability distribution over a finite set $\mathcal{Z}$ of signal realizations conditional on the true preference shock being θ. For each signal realization $z \in \mathcal{Z}$, the student submits a rank-order list that determines her matching outcome. The cost of information acquisition is $\mu \cdot I(\Pi)$, where $\mu \geq 0$ is the marginal cost of information, and $I(\Pi)$ is the mutual information between the preference shock and the signals generated by Π. Specifically, $I(\Pi) = H(\theta) - \mathbb{E}_\Pi [H(\theta \mid z)]$, where $H(\cdot)$ denotes the Shannon entropy of a random variable.

It is without loss of generality to consider signal structures that recommend students to submit either sab or asb, and students strictly follow these recommendations (Matějka and McKay, 2015).[3] Accordingly, we represent the signal acquired by a student by an integrable function $m : [0, 1] \to [0, 1]$, where $m(\theta)$ specifies the probability that the student is recommended to submit sab when her unobservable preference shock is $\theta \in [0, 1]$. Define $\overline{m} \equiv \int_0^1 m(\theta)\, d\theta$ as the average probability of the recommendation

[3] The idea is that, given a general signal structure, signal realizations inducing sab can be merged into a single recommendation, and likewise for those inducing asb. This merge results in the same matching payoffs at lower acquisition costs, since the new signal structure is less Blackwell informative.

to submit *sab*. The information cost is $\mu \cdot I(m)$, where $\mu \geq 0$ and

$$I(m) = \int_0^1 [m(\theta) \ln m(\theta) + (1 - m(\theta)) \ln(1 - m(\theta))] d\theta \\ - \overline{m} \ln \overline{m} - (1 - \overline{m}) \ln(1 - \overline{m}). \quad (1)$$

2.1 Matching mechanisms

We examine two mechanisms: the Immediate-Acceptance (Boston) mechanism and the Deferred-Acceptance (DA) mechanism. We assume a single tie-breaking rule in which student priorities are drawn independently from a uniform distribution over $[0, 1]$ at the beginning of the mechanism and are used by all schools. A student with a higher priority score is ranked higher, and students do not observe their priorities when submitting rank-order lists.

The Boston mechanism assigns students to schools in multiple rounds. In each round, unmatched students apply to their top remaining choice, and schools admit students in order of priority until capacity is reached. The rounds proceed as follows in our setup. Let $r \in [0, 1]$ denote the proportion of students who submit *sab*, and suppose $r \geq 1 - \lambda_a$. In the first round, school s receives more applications than its capacity ($r \geq 1 - \lambda_a > \lambda_s$), so it admits the λ_s highest-priority applicants among the r students submitting *sab* and rejects the remaining $r - \lambda_s$. Meanwhile, all students who submitted *asb* are matched with school a. School s has reached its capacity, while school a still has $\lambda_a - (1 - r)$ remaining seats. In the second round, students who submitted *sab* but were rejected by school s apply to school a, which admits the $\lambda_a - (1 - r)$ highest-priority applicants until its capacity is reached. The remaining students are matched with school b in the third round. Table 1 reports the final allocation under all cases; derivations for the remaining cases are omitted for brevity.

	s	a	b
sab	λ_s	$\lambda_a - (1 - r)$	λ_b
asb	0	$1 - r$	0

(a) if $r \geq 1 - \lambda_a$

	s	a	b
sab	λ_s	0	$r - \lambda_s$
asb	0	λ_a	$(1 - r) - \lambda_a$

(b) if $\lambda_s \leq r \leq 1 - \lambda_a$

	s	a	b
sab	r	0	0
asb	$\lambda_s - r$	λ_a	λ_b

(c) if $r \leq \lambda_s$

Table 1: Final allocation of students to schools under the Boston mechanism. Each cell reports the mass of students who submitted the rank-order list in the corresponding row and were matched with the school in the corresponding column.

The DA mechanism proceeds similarly to the Boston mechanism, but defers all tentative assignments until the end of the final round. The mechanism can equivalently be characterized by market-clearing priority cutoffs (Azevedo and Leshno, 2016). Market-clearing cutoffs $p_s, p_a, p_b \in [0, 1]$ are such that assigning each student to her top choice among schools whose cutoff falls below her priority exactly meets each school's capacity. It is known that such cutoffs exist and are unique. In our setup, every student is guaranteed to match with at least school b, so $p_b = 0$. If a sufficiently large fraction $r \geq \frac{\lambda_s}{\lambda_s+\lambda_a}$ of students report *sab* — that is, school s receives more applications than school a relative to their capacities — then the market-clearing cutoffs satisfy $p_s \geq p_a > p_b = 0$.

The assignment of students under the DA mechanism is determined as follows. Students reporting *sab* are assigned to school s if their priority exceeds p_s, to school a if their priority falls between p_a and p_s, and to school b if their priority falls below p_a. Students reporting *asb* are assigned to school a or b depending on whether their priority exceeds or falls below p_a. The resulting assignment by priority cutoffs is:

	s	a	b
sab	$(1-p_s)r$	$(p_s-p_a)r$	$p_a r$
asb	0	$(1-p_a)(1-r)$	$p_a(1-r)$

.

The cutoffs are market-clearing if $(1-p_s)r = \lambda_s$ and $(p_s - p_a)r + (1-p_a)(1-r) = \lambda_a$, subject to $p_s \geq p_a > p_b = 0$. Substituting the market-clearing cutoffs yields the student assignment shown in Panel (a) of Table 2. The remaining case is omitted for brevity.[4]

	s	a	b
sab	λ_s	$r\lambda_a - (1-r)\lambda_s$	$\lambda_b r$
asb	0	$(1-r)(\lambda_s+\lambda_a)$	$\lambda_b(1-r)$

(a) if $r \geq \frac{\lambda_s}{\lambda_s+\lambda_a}$

	s	a	b
sab	$r(\lambda_s+\lambda_a)$	0	$r\lambda_b$
asb	$(1-r)\lambda_s - r\lambda_a$	λ_a	$(1-r)\lambda_b$

(b) if $r < \frac{\lambda_s}{\lambda_s+\lambda_a}$

Table 2: Final allocation of students to schools under the DA mechanism. Each cell reports the mass of students who submitted the rank-order list in the corresponding row and were matched with the school in the corresponding column.

[4]The Top-Trading-Cycles (TTC) mechanism with a random endowment is equivalent to the DA mechanism in our setting. For any proportion r of students reporting *sab*, the initial assignment is

	s	a	b
sab	$r\lambda_s$	$r\lambda_a$	$r\lambda_b$
asb	$(1-r)\lambda_s$	$(1-r)\lambda_a$	$(1-r)\lambda_b$

.

Students reporting *sab* but assigned to school a then trade with those reporting *asb* but assigned to school s, yielding the same assignment as in Panel (a) of Table 2. For a characterization of the TTC mechanism in a general continuum environment, see Leshno and Lo (2021).

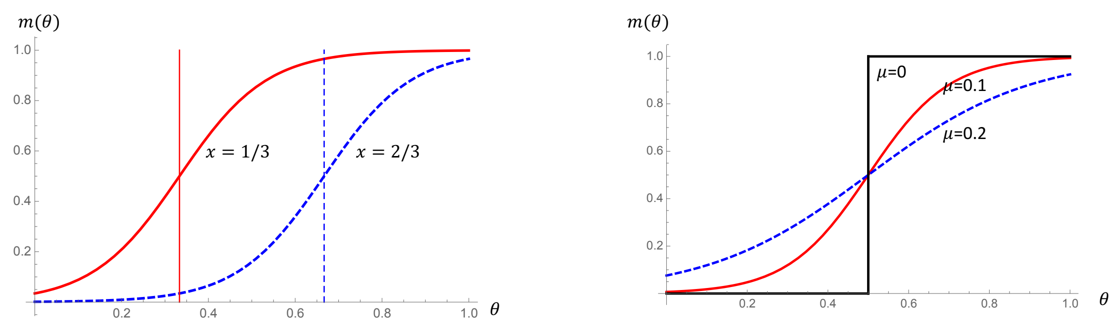
2.2 Information acquisition

Each mechanism $\Gamma \in \{\text{Boston}, \text{DA}\}$ defines a game in which each student must acquire information about her unobservable preference shock θ at a cost, and subsequently submit a rank-order list, either sab or asb, to the mechanism.

A student's problem is defined as follows. Given a mechanism Γ and a fraction r of students reporting sab, let $U^{\Gamma}_{sab}(\theta; r)$ and $U^{\Gamma}_{asb}(\theta; r)$ denote the expected match payoffs of a student with preference type θ from submitting sab and asb, respectively. Let $\Delta^{\Gamma}(\theta; r) \equiv U^{\Gamma}_{sab}(\theta; r) - U^{\Gamma}_{asb}(\theta; r)$ denote the net payoff gain from submitting sab over asb. The student then solves

$$\max_{m:[0,1]\to[0,1]} \int_0^1 m(\theta)\Delta^{\Gamma}(\theta; r)d\theta - \mu I(m). \tag{2}$$

An optimal signal structure captures both what and how much a student should learn about her preference type. In Figure 1, we illustrate an optimal signal structure when $\Delta(\theta) = \theta - x$ for $x \in (0, 1)$. The optimal signal $m(\theta)$ is increasing in θ



(a) Learning focus ($\mu = 0.1$). (b) Learning intensity ($x = 1/2$).

Figure 1: An optimal information acquisition strategy when $\Delta(\theta) = \theta - x$.

because the payoff gain $\Delta(\theta)$ from switching from asb to sab increases with θ. The student therefore aims to learn whether her preference shock θ falls above or below the threshold x, choosing sab if and only if $\theta > x$, as shown in Panel (a). As the marginal cost of information μ decreases, the student acquires more information, and the recommendation $m(\theta)$ responds more precisely to θ, as shown in Panel (b).

3 Equilibrium Analysis

A game is defined by a mechanism $\Gamma \in \{\text{Boston}, \text{DA}\}$. We characterize a symmetric Nash equilibrium m^Γ of this game, where $r^\Gamma \equiv \int m^\Gamma(\theta)\, d\theta$ denotes the equilibrium proportion of students reporting *sab*. In equilibrium, m^Γ is the unique solution to each student's information acquisition problem (Equation 2) given that a fraction r^Γ of other students report *sab*. It therefore suffices to find the equilibrium proportion r^Γ.

We focus on parameter values under which the equilibrium proportion of students submitting *sab* exceeds that of *asb*, relative to schools' capacities. Specifically, given capacity profile $(\lambda_s, \lambda_a, \lambda_b)$ and marginal information cost μ, we assume v is sufficiently large to ensure that school s is always more competitive than school a in equilibrium.[5] Precisely, the equilibrium fraction of students submitting *sab* exceeds $\frac{\lambda_s}{\lambda_s+\lambda_a}$ under both mechanisms, so that under the Boston mechanism school s fills up no later than school a, and under the DA mechanism the priority cutoff for school s is weakly higher than for school a.

For the Boston mechanism, we consider two cases: $r \geq 1-\lambda_a$ (Panel (a) of Table 1) and $\frac{\lambda_s}{\lambda_s+\lambda_a} < r < 1-\lambda_a$ (Panel (b) of Table 1). When $r \geq 1-\lambda_a$, a student submitting *sab* is assigned to schools s, a, and b with probabilities $\frac{\lambda_s}{r}$, $\frac{\lambda_a-(1-r)}{r}$, and $\frac{\lambda_b}{r}$, respectively, while a student submitting *asb* is assigned to school a with certainty. A similar analysis applies to the case $\frac{\lambda_s}{\lambda_s+\lambda_a} < r < 1-\lambda_a$. The expected payoff gain $\Delta^B(\theta; r)$ from submitting *sab* over *asb*, for any fraction $r > \frac{\lambda_s}{\lambda_s+\lambda_a}$ of students submitting *sab* and preference type θ, is

$$\Delta^B(\theta; r) = \frac{\lambda_s(v+\theta)}{r} - \min\left\{\frac{\lambda_a}{1-r}, \frac{1-\lambda_a}{r}\right\}, \quad \forall r > \frac{\lambda_s}{\lambda_s+\lambda_a}. \tag{3}$$

For the DA mechanism (Panel (a) of Table 2), a student reporting *sab* is assigned to schools s, a, and b with probabilities $\frac{\lambda_s}{r}$, $\lambda_a - \frac{(1-r)\lambda_s}{r}$, and λ_b, respectively, while a student reporting *asb* is assigned to schools s, a, and b with probabilities 0, $\lambda_s+\lambda_a$, and λ_b. This implies that

$$\begin{aligned}\Delta^D(\theta; r) &= \left(\frac{\lambda_s}{r}(v+\theta) + \lambda_a - \frac{1-r}{r}\lambda_s\right) - (\lambda_s+\lambda_a) \\ &= \frac{\lambda_s(v+\theta)}{r} - \frac{\lambda_s}{r}, \qquad \forall r > \frac{\lambda_s}{\lambda_s+\lambda_a}. \end{aligned} \tag{4}$$

[5] When the marginal cost μ is very high, assuming $v > 1/2$ is sufficient, as most students choose *sab* based on ex-ante utilities, $E[u_s] = v + E[\theta] > u_a$. When μ is low, the required condition on v depends on the schools' capacities. A school preferred by the majority of students, $\Pr[v+\theta > u_a] = v > 1/2$, may be less selective if its capacity is sufficiently large.

When a large proportion of students apply to school s first ($r > \frac{\lambda_s}{\lambda_s+\lambda_a}$), submitting sab entails greater risk under the Boston mechanism than under the DA mechanism:

Lemma 1. *For any $r > \frac{\lambda_s}{\lambda_s+\lambda_a}$ and $\theta \in [0,1]$, $\Delta^D(\theta; r) > \Delta^B(\theta; r)$.*

Proof. We have $\Delta^D - \Delta^B = \min\left\{\frac{\lambda_a}{1-r}, \frac{1-\lambda_a}{r}\right\} - \frac{\lambda_s}{r}$. Since $\frac{1-\lambda_a}{r} = \frac{\lambda_b+\lambda_s}{r} > \frac{\lambda_s}{r}$, and $r > \frac{\lambda_s}{\lambda_s+\lambda_a}$ implies $\frac{\lambda_a}{1-r} > \frac{\lambda_s}{r}$, both terms in the minimum exceed $\frac{\lambda_s}{r}$, completing the proof. □

The intuition is straightforward. Under the Boston mechanism, failing to match with school s in the first round is likely to trigger another rejection in the second round, potentially resulting in a match with the worst school b. Under the DA mechanism, by contrast, choosing sab does not increase the risk of being matched with school b.

3.1 A benchmark case of free information acquisition

A standard model in which students know their own preferences corresponds to the case of $\mu = 0$. For any mechanism $\Gamma \in \{\text{Boston}, \text{DA}\}$, an equilibrium strategy takes the form

$$m^\Gamma(\theta) = \begin{cases} 1 & \text{if } \theta > \theta^\Gamma \\ 0 & \text{if } \theta < \theta^\Gamma, \end{cases}$$

with a mechanism-dependent threshold θ^Γ. Any value in $[0,1]$ at the threshold θ^Γ can be chosen in equilibrium. Under the DA mechanism with single tie-breaking, a student's dominant strategy is to use the threshold $\theta^D = 1 - v$, so that the proportion of students reporting sab is $r^D = 1 - \theta^D = v$, which is assumed to exceed $\frac{\lambda_s}{\lambda_s+\lambda_a}$. For the Boston mechanism, the equilibrium threshold θ^B is the unique solution to $\Delta^B(\theta; r^B) = 0$, where $r^B = 1 - \theta^B > \frac{\lambda_s}{\lambda_s+\lambda_a}$.[6]

Under the Boston mechanism, a student submits sab only if her preference shock θ is substantially greater than $\theta^D = 1 - v$:

Lemma 2. *If $v > \frac{\lambda_s}{\lambda_s+\lambda_a}$, then $\theta^B > \theta^D$.*

Proof. By Lemma 1, for any $r > \frac{\lambda_s}{\lambda_s+\lambda_a}$, we have $\Delta^B(\theta^D; r) < \Delta^D(\theta^D; r) = 0$, where $\theta^D = 1 - v$. Since $\Delta^B(\theta; r)$ is strictly increasing in θ, the unique solution θ^B to $\Delta^B(\theta; r^B) = 0$, where $r^B = 1 - \theta^B > \frac{\lambda_s}{\lambda_s+\lambda_a}$, must satisfy $\theta^B > \theta^D$. □

Students who are nearly indifferent between schools s and a, with preference shocks $\theta \in (\theta^D, \theta^B)$, are deterred from submitting sab under the Boston mechanism due to

[6] The unique solution θ^B satisfies either (i) $\theta^B \le \lambda_a$ and $\lambda_s(v + \theta^B) = 1 - \lambda_a$, or (ii) $\lambda_a \le \theta^B \le 1 - \lambda_s$ and $\frac{\lambda_s(v+\theta^B)}{1-\theta^B} = \frac{\lambda_a}{\theta^B}$.

the risk of multiple rejections. Under the DA mechanism, by contrast, such students submit *sab* since the mechanism is strategy-proof. Consequently, a larger proportion of students under the DA mechanism rank school s first, resulting in more homogeneous rank-order reports.

The homogeneous reports cause the DA mechanism to rely more heavily on random tie-breaking, giving rise to an efficiency loss. We formalize this inefficiency as follows. Every allocation assigns students to schools a and b up to capacity, and all students assigned to either school receive the same match payoff, $u_a = 1$ or $u_b = 0$. Hence, the efficiency of an allocation can be evaluated based on the idiosyncratic preferences of students assigned to school s.

Consider an allocation represented by a density function $g(\theta)$ of the idiosyncratic preference shock θ among students assigned to school s, satisfying $\int g(\theta)\, d\theta = \lambda_s$. We say that allocation g is *more efficient* than allocation g' if g first-order stochastically dominates g':

$$\int_0^{\overline{\theta}} g(\theta)\, d\theta \leq \int_0^{\overline{\theta}} g'(\theta)\, d\theta$$

for every $\overline{\theta} \in (0,1)$, with strict inequality for at least one $\overline{\theta} \in (0,1)$.

Corollary 1. *In the case of free information acquisition, the DA mechanism is less efficient than the Boston mechanism.*

Proof. In equilibrium, school s is more competitive than school a ($r^\Gamma > \frac{\lambda_s}{\lambda_s+\lambda_a}$) under any mechanism $\Gamma \in \{\text{Boston}, \text{DA}\}$, so a student can match with school s only by submitting *sab*. A student with preference shock θ is therefore assigned to school s with probability $g^\Gamma(\theta) \equiv m^\Gamma(\theta)\frac{\lambda_s}{r^\Gamma}$, where $m^\Gamma(\theta)$ is the probability of submitting *sab* given preference shock θ, and $\frac{\lambda_s}{r^\Gamma}$ is the probability of matching with school s conditional on submitting *sab*. In equilibrium, the allocation under each mechanism $\Gamma \in \{\text{Boston}, \text{DA}\}$ is

$$g^\Gamma(\theta) = \begin{cases} \frac{\lambda_s}{1-\theta^\Gamma} & \text{if } \theta > \theta^\Gamma, \\ 0 & \text{if } \theta < \theta^\Gamma. \end{cases}$$

The result then follows directly from $\theta^B > \theta^D$ (Lemma 2). □

3.2 Equilibrium under costly information acquisition

We continue to consider parameter values such that, for each mechanism $\Gamma \in \{\text{Boston}, \text{DA}\}$, a symmetric Nash equilibrium $m^\Gamma(\theta)$ exists with the proportion of students submitting *sab* in $\left(\frac{\lambda_s}{\lambda_s+\lambda_a}, 1\right)$. That is, v is sufficiently large to make school s more competitive than school a, but not so large as to preclude an interior equilibrium.[7]

Given a mechanism $\Gamma \in \{\text{Boston}, \text{DA}\}$, suppose that a proportion r^Γ of other students report *sab*. If the information acquisition problem (Equation 2) has an interior solution $m(\theta)$, it satisfies the first-order condition (Yang, 2015):

$$\Delta^\Gamma(\theta; r^\Gamma) = \mu \cdot \left[\ln\left(\frac{m(\theta)}{1-m(\theta)}\right) - \ln\left(\frac{\overline{m}}{1-\overline{m}}\right)\right], \quad \forall\theta \in [0,1], \tag{5}$$

where $\overline{m} \equiv \int m(\theta)\, d\theta \in (0,1)$. In a symmetric Nash equilibrium, $r^\Gamma = \overline{m}$, so (5) yields

$$m(\theta) = \left(1 + \frac{1-r^\Gamma}{r^\Gamma} \exp\left(-\frac{\Delta^\Gamma(\theta; r^\Gamma)}{\mu}\right)\right)^{-1}, \quad \forall\theta \in [0,1], \tag{6}$$

where r^Γ solves $r^\Gamma = \int m(\theta)\, d\theta$.

Lemma 3. *An information acquisition strategy $m(\cdot)$ with $\overline{m} \in \left(\frac{\lambda_s}{\lambda_s+\lambda_a}, 1\right)$ is a symmetric Nash equilibrium under mechanism $\Gamma \in \{\text{Boston}, \text{DA}\}$ if and only if it takes the form of* (6)*, where $r^\Gamma \in \left(\frac{\lambda_s}{\lambda_s+\lambda_a}, 1\right)$ is a solution to*

$$\exp\left(\frac{\lambda_s}{\mu}\right) = 1 + \frac{\exp\left(\frac{\lambda_s}{r\mu}\right) - 1}{\frac{1-r}{r}\exp\left(-\frac{\Delta^\Gamma(0;r)}{\mu}\right) + 1}. \tag{7}$$

Therefore, it remains to find the proportion of students submitting *sab*, i.e., $r^\Gamma \in \left(\frac{\lambda_s}{\lambda_s+\lambda_a}, 1\right)$ in equilibrium. A careful inspection of (7) yields:

Proposition 1. *For any $\mu > 0$,*

1. ***DA equilibrium:*** *There exist $\underline{v}, \overline{v} \in (0,1)$ with $\underline{v} < \overline{v}$ such that a unique interior equilibrium of the DA mechanism with $r^D \in \left(\frac{\lambda_s}{\lambda_s+\lambda_a}, 1\right)$ exists if and only if $v \in (\underline{v}, \overline{v})$.*
2. ***Boston equilibrium:*** *Whenever a DA interior equilibrium exists, the Boston mechanism also has a unique interior equilibrium with $r^B \in \left(\frac{\lambda_s}{\lambda_s+\lambda_a}, r^D\right)$.*

The second part of Proposition 1 extends Lemma 2 to the case of costly information acquisition: a larger proportion of students submit *sab* under the DA mechanism than under the Boston mechanism, reflecting greater willingness to apply to the competitive school s first.

The proof of Proposition 1 follows from a careful analysis of (7). The left-hand side (LHS) is constant at $\exp\left(\frac{\lambda_s}{\mu}\right)$. Under the DA mechanism, the right-hand side

[7] If v is very close to 1, a corner equilibrium arises in which students acquire no information under the DA mechanism and the equilibrium allocation becomes independent of preference types.

(RHS) equals the LHS at $r = 1$ and increases without bound as $r \to 0$. The RHS is strictly single-dipped in r.[8] By the intermediate value theorem, a unique solution to (7) exists in $(0, 1]$. Since the RHS is strictly increasing in v for each fixed $r < 1$, one can identify bounds $\underline{v}$ and $\overline{v}$ on v that ensure the solution lies in $\left(\frac{\lambda_s}{\lambda_s+\lambda_a}, 1\right)$. For the Boston mechanism, $\Delta^D(0, r) > \Delta^B(0, r)$ for every $r > \frac{\lambda_s}{\lambda_s+\lambda_a}$ (Lemma 1), so the RHS under the Boston mechanism is strictly smaller than under the DA mechanism for every $r \in \left(\frac{\lambda_s}{\lambda_s+\lambda_a}, 1\right)$, with the two coinciding at $r = \frac{\lambda_s}{\lambda_s+\lambda_a}$ and $r = 1$. It follows that $r^B \in \left(\frac{\lambda_s}{\lambda_s+\lambda_a}, r^D\right)$.

3.3 The DA mechanism's efficiency loss

Recall that we measure allocation efficiency by the distribution of preference shocks among students assigned to school s, denoted by $g^\Gamma(\theta)$ for each mechanism $\Gamma \in \{\text{Boston}, \text{DA}\}$.

Proposition 2. *Suppose an interior equilibrium with $r^D \in \left(\frac{\lambda_s}{\lambda_s+\lambda_a}, 1\right)$ exists for the DA mechanism, which implies the existence of an interior equilibrium with $r^B \in \left(\frac{\lambda_s}{\lambda_s+\lambda_a}, r^D\right)$ for the Boston mechanism. Then g^B single-crosses g^D from below.*[9]

Corollary 2. *The DA mechanism yields a lower allocation efficiency than the Boston mechanism:*

$$\int_0^{\overline{\theta}} g^D(\theta)\, d\theta > \int_0^{\overline{\theta}} g^B(\theta)\, d\theta, \quad \forall \overline{\theta} \in (0, 1).$$

We illustrate Proposition 2 and Corollary 2 in Figure 2 for the parameter values $v = 0.6$, $\mu = 0.1$, and $\lambda_j = 1/3$ for $j \in \{s, a, b\}$. The horizontal line at λ_s represents a purely random assignment of students to school s. The red solid line represents the equilibrium allocation density g^D under the DA mechanism. The graph crosses the λ_s line at $\theta = 1 - v$, the preference shock at which a student is indifferent between reporting sab and asb. The blue dashed line represents the equilibrium allocation density g^B under the Boston mechanism. Since submitting sab under the Boston mechanism entails a higher risk of matching with school b, a student requires a strictly higher preference shock $\theta > 1 - v$ to be indifferent between sab and asb. Consequently, g^B single-crosses g^D from below, confirming the inefficiency of the DA mechanism.

We now examine the sources of the DA mechanism's efficiency loss relative to the Boston mechanism. Even under free information acquisition, students submit more

[8] A single-dipped function is the negative of a single-peaked function.

[9] That is, for all $\theta' < \theta''$, if $g^B(\theta') \geq g^D(\theta')$, then $g^B(\theta'') > g^D(\theta'')$.

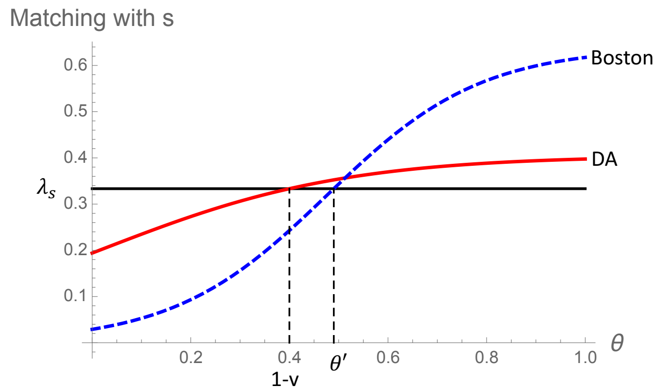


Figure 2: Equilibrium allocation probabilities of matching with school s under the DA and Boston mechanisms, with parameter values $v = 0.6$, $\mu = 0.1$, and $\lambda_j = 1/3$ for $j \in \{s, a, b\}$.

homogeneous rank-order lists under the DA mechanism, increasing its reliance on tie-breaking. Once information becomes costly, this homogeneity discourages students from learning their idiosyncratic preference shocks. As students acquire less information, their rank-order reports become less responsive to preference shocks, further homogenizing submissions. This in turn weakens incentives for information acquisition, creating a self-reinforcing cycle.

We derive the DA mechanism's inefficiency in two steps. The first captures students' myopic responses to a switch from the Boston to the DA mechanism. The second demonstrates the feedback loop between homogeneous rank-order reports and reduced incentives for information acquisition through their subsequent myopic responses.

The first step characterizes students' responses to a switch from the Boston to the DA mechanism, holding their beliefs about others' strategies fixed at $r^B \in \left(\frac{\lambda_s}{\lambda_s+\lambda_a}, 1\right)$. Specifically, for any $r \in [r^B, r^D]$, let $m^H(\cdot; r, \mu)$ denote the interior optimal strategy when a student believes that a proportion r of peers will report *sab* to the DA mechanism. The efficiency of the resulting allocation is then $g^H(\theta; r, \mu) \equiv m^H(\theta; r, \mu)\frac{\lambda_s}{\overline{m}^H(r,\mu)}$, where $\overline{m}^H(r, \mu) \equiv \int m^H(\theta; r, \mu)\, d\theta$.

Proposition 3. *Take $v \in (\underline{v}, \overline{v})$ such that the DA and Boston mechanisms have interior equilibria with $\frac{\lambda_s}{\lambda_s+\lambda_a} < r^B < r^D < 1$. Let $m^H(\cdot; r^B, \mu)$ be the optimal strategy when the mechanism switches from Boston to DA while students' beliefs about the proportion of others reporting sab are held fixed at r^B.*

Then, (i) $\overline{m}^H(r^B, \mu) > r^B$, and (ii) the allocation distribution g^B single-crosses $g^H(\cdot; r^B, \mu)$ from below, implying that the allocation becomes less efficient following the mechanism switch.

To understand Proposition 3, suppose students believe that the probability of matching with school s when submitting sab remains unchanged at $\frac{\lambda_s}{r^B}$ from the Boston mechanism. The switch to the DA mechanism nevertheless eliminates the risk of a bad match from submitting sab. Consequently, a larger proportion of students choose to submit sab (Part (i)), leading to greater reliance on random tie-breaking in assigning school s, which in turn reduces allocation efficiency (Part (ii)).

The second step demonstrates the feedback loop between homogeneous rank-order submissions and reduced information acquisition:

Proposition 4. *Suppose $\mu < \lambda_s(1-v)$ so that $m^H(\cdot; r, \mu)$ is an interior optimal strategy under the DA mechanism for any $r \in \left(\frac{\lambda_s}{\lambda_s+\lambda_a}, 1\right)$. Then, for any $r_1, r_2 \in (r^B, r^D)$ with $r_1 < r_2$, we have $r^B < \overline{m}^H(r_1, \mu) < \overline{m}^H(r_2, \mu) < r^D$, and the allocation distribution $g^H(\cdot; r_1, \mu)$ single-crosses $g^H(\cdot; r_2, \mu)$ from below, implying that the allocation is less efficient under the higher proportion r_2.*

In words, when students expect more homogeneous rank-order submissions, the mechanism relies more heavily on random tie-breaking, weakening their incentive to learn idiosyncratic preference shocks. This in turn renders rank-order reports less sensitive to unobservable preference shocks, further homogenizing submissions.

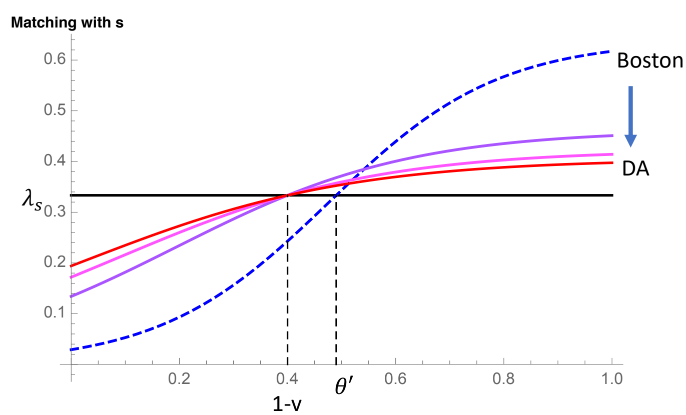


Figure 3: Allocation efficiency declines following the switch from the Boston to the DA mechanism and subsequent best-response adjustments. Parameter values are $v = 0.6$, $\mu = 0.01$, and $\lambda_j = 1/3$ for $j \in \{s, a, b\}$.

Figure 3 illustrates the implications of Proposition 3 and Proposition 4 in terms of allocation efficiency. The blue dashed line represents the equilibrium allocation under the Boston mechanism. When the mechanism switches to the DA but students best-respond to r^B, their rank-order submissions become more homogeneous, particularly for students with unobservable preference types near $1 - v$. The subsequent transitions, leading ultimately to the DA equilibrium shown by the red solid line, trace the self-reinforcing cycle between more homogeneous rank-order submissions and reduced information acquisition. Throughout this sequence of adjustments, allocations become increasingly inefficient.

4 Monotonicity of DA Efficiency Loss in Information Costs

The self-reinforcing cycle between homogeneous rank-order submissions and reduced information acquisition is intensified as the information cost increases. As μ increases, the optimal information strategy $m(\theta; r, \mu)$ becomes less sensitive to the idiosyncratic preference type: $m(\theta; r, \mu)$ increases in μ for any $\theta < 1 - v$ and decreases in μ for any $\theta > 1 - v$ (Equation 7). This leads to more homogeneous rank-order reports and greater efficiency loss under the DA mechanism:[10]

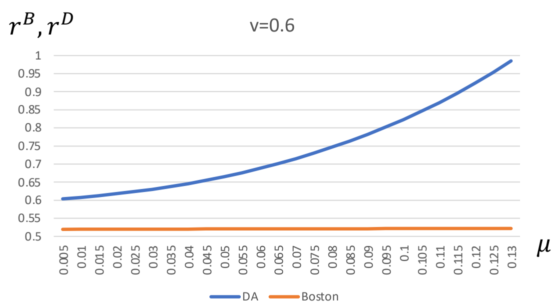
Proposition 5. *Suppose $\mu_1 < \mu_2 < \lambda_s(1-v)$ so that $m^H(\cdot; r, \mu)$ is an interior optimal strategy under the DA mechanism for $\mu \in \{\mu_1, \mu_2\}$ and any $r \in \left(\frac{\lambda_s}{\lambda_s+\lambda_a}, 1\right)$. Then, the allocation distribution $g^H(\cdot; r, \mu_1)$ single-crosses $g^H(\cdot; r, \mu_2)$ from below, implying that the allocation is less efficient under the higher information cost μ_2.*

We illustrate that the DA mechanism's efficiency loss increases in the information costs using the average efficiency:

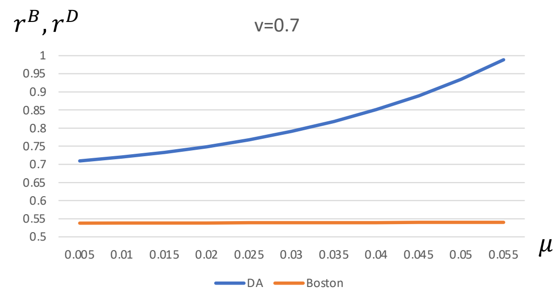
$$W^\Gamma \equiv \frac{\int_0^1 \theta g^\Gamma(\theta) d\theta}{\lambda_s}, \quad \text{for } \Gamma \in \{\text{Boston}, \text{DA}\}.$$

Figure 4 illustrates the monotonicity of the DA mechanism's efficiency loss in the information cost μ. Panels (a) and (b) show that as μ increases, the proportion of students reporting *sab* rapidly approaches 1 under the DA mechanism while remaining low under the Boston mechanism. While Proposition 6 in the Appendix establishes this result for general capacity profiles and parameter values, the figures present numerical examples with equal capacities ($\lambda_s = \lambda_a = \lambda_b = 1/3$), two values of v ($v = 0.6$ and $v = 0.7$), and values of μ ranging from 0 upward while ensuring interior equilibria under both mechanisms. Panel (c) shows that the efficiency gap $W^B - W^D$ widens as μ increases, using the same numerical examples.

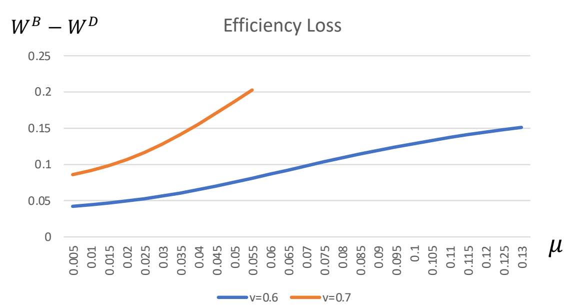
[10] Recall that $m^H(\cdot; r, \mu)$ denotes the optimal information acquisition strategy under the DA mechanism at information cost μ when a student believes that a proportion r of other students submit *sab*.



(a) $v = 0.6$



(b) $v = 0.7$



(c) The DA mechanism's efficiency loss.

Figure 4: As μ increases, the homogeneity of rank-order reports to the DA mechanism increases rapidly, and results in an increasing efficiency loss.

5 Conclusion

We analyze a school choice problem in which students acquire information about their preferences before submitting rank-order lists to a mechanism. We compare two widely used mechanisms: the DA and Boston mechanisms. The DA mechanism induces more homogeneous rank-order submissions, causing it to rely more heavily on random tie-breaking than the Boston mechanism, giving rise to the DA mechanism's inefficiency.

While this source of inefficiency has been well-known, we find that it is further amplified when students endogenously choose to learn their preferences. Greater reliance on tie-breaking weakens students' incentives to acquire costly information, leading to further homogenization of rank-order submissions and greater efficiency loss. This self-reinforcing cycle is more pronounced when the marginal cost of information acquisition is higher.

A Appendix

A.1 Proof of Lemma 3

For a given mechanism $\Gamma \in \{\text{Boston}, \text{DA}\}$, the equilibrium consistency $r^\Gamma = \int_0^1 m^\Gamma(\theta)d\theta$, where m^Γ satisfies (6), implies that r^Γ is a solution to the following equation:

$$r = \int_0^1 \left(1 + \frac{1-r}{r} \exp\left(-\frac{\Delta^\Gamma(\theta; r)}{\mu}\right)\right)^{-1} d\theta.$$

Since $\Delta^\Gamma(\theta; r) = \Delta^\Gamma(0; r) - \frac{\lambda_s \theta}{r}$ (as shown in (3) and (4)), we can rewrite the above equation as follows:

$$\begin{aligned} r &= \int_0^1 \left(1 + \frac{1-r}{r} \exp\left(-\frac{\Delta^\Gamma(0; r)}{\mu}\right) \exp\left(\frac{\lambda_s \theta}{r\mu}\right)\right)^{-1} d\theta \\ &= \frac{r\mu}{\lambda_s} \log\left[\frac{1-r}{r} \exp\left(-\frac{\Delta^\Gamma(0; r)}{\mu}\right) + \exp\left(\frac{\lambda_s \theta}{r\mu}\right)\right]\Bigg|_0^1. \end{aligned}$$

By canceling out r and taking the exponential of both sides, we obtain (7).

A.2 Proof of Proposition 1

A.2.1 Proof for the DA mechanism

Given any $\mu > 0$, we determine the bounds $\underline{v}$ and $\overline{v}$ such that $v \in (\underline{v}, \overline{v})$ if and only if there exists a solution r to the equilibrium condition (7) under the DA mechanism in the interval $(\hat{r}, 1)$, where $\hat{r} \equiv \frac{\lambda_s}{\lambda_s + \lambda_a}$.

First, we find the upper bound $\overline{v}$. Let $z \equiv \frac{\lambda_s}{\mu}$, $w \equiv \frac{\lambda_a}{\mu}$, and $x \equiv \frac{1}{r} \in [1, \infty)$. The equilibrium condition (7) becomes:

$$e^z = 1 + \frac{e^{zx} - 1}{(x-1)e^{zx(1-v)} + 1} \iff f(x, v) \equiv \frac{e^{zx} - 1}{e^z - 1} - (x-1)e^{zx(1-v)} = 1.$$

For any v, we have $f(1; v) = 1$ and $\lim_{x \to \infty} f(x, v) = \infty$. Moreover,

$$\begin{aligned} \frac{\partial f(x, v)}{\partial x} < 0 &\iff \frac{ze^{zx}}{e^z - 1} - e^{zx(1-v)} - (x-1)e^{zx(1-v)} z(1-v) < 0 \\ &\iff g(x, v) \equiv \frac{ze^{zxv}}{e^z - 1} - 1 - (x-1)z(1-v) < 0. \end{aligned} \tag{8}$$

Let $\overline{v}$ be defined by:

$$g(1,\overline{v}) = 0 \iff ze^{z\overline{v}} = e^z - 1 \iff \overline{v} = \frac{1}{z}\log\left(\frac{e^z-1}{z}\right). \tag{9}$$

One can verify that $\overline{v} \in (1/2, 1)$ and is strictly increasing in z. These properties will be used below.

If $v < \overline{v}$, then $g(1,v) < 0$ since g is strictly increasing in v and $g(1,\overline{v}) = 0$. Moreover, $g(x,v)$ is strictly convex in x with $\lim_{x\to\infty} g(x,v) = \infty$. Thus, as x increases from 1 to ∞, $g(x,v)$ changes sign exactly once from strictly negative to strictly positive, implying that $f'(x,v) < 0$ at $x = 1$ and $f(x,v)$ is strictly single-dipped in x. Hence, a unique solution $x^* > 1$ to $f(x;v) = 1$ exists (i.e., a unique equilibrium proportion $r^* = \frac{1}{x^*} < 1$), and therefore a unique solution r^* to (7) exists in $(0,1)$.

If $v \geq \overline{v}$, then $g(1;v) \geq 0$. Moreover, for any $x > 1$,

$$\frac{\partial g(x,v)}{\partial x} > \frac{\partial g(1,v)}{\partial x} \geq (z\overline{v})\frac{ze^{z\overline{v}}}{e^z-1} - z(1-\overline{v}) \geq z(2\overline{v}-1) \geq 0, \tag{10}$$

where the last two inequalities follow from (9) and $\overline{v} \geq 1/2$, respectively. Thus, $g(x;v) > 0$ for every $x > 1$, and no solution $x > 1$ to $f(x;v) = 1$ exists.

Second, we find the lower bound $\underline{v}$. Let $\hat{x} \equiv \frac{1}{\hat{r}} = \frac{\lambda_s+\lambda_a}{\lambda_s} > 1$ and define $\underline{v}$ by

$$\begin{aligned} f(\hat{x};\underline{v}) = 1 &\iff \frac{e^{z+w}-1}{e^z-1} - \frac{w}{z}e^{(z+w)(1-\underline{v})} = 1 \iff e^{(z+w)\underline{v}} = \frac{w}{1-e^{-w}}\frac{e^z-1}{z} \\ &\iff \underline{v} = \frac{1}{z+w}\log\left(\frac{w}{1-e^{-w}}\frac{e^z-1}{z}\right). \end{aligned}$$

Recall that $\overline{v}$ was defined by $f(1;\overline{v}) = 1$. Since (8) holds with opposite inequalities and $g(x,\overline{v}) > 0$ for every $x > 1$ (as we observed after (10)), we have $\frac{\partial f(x,\overline{v})}{\partial x} > 0$ for every $x > 1$. Thus, $f(\hat{x},\overline{v}) > 1 = f(\hat{x},\underline{v})$, which implies $\overline{v} > \underline{v}$.

We have shown that $v < \overline{v}$ if and only if a unique solution $x^* > 1$ to $f(x,v) = 1$ exists. Since $f(x,v)$ is strictly and continuously increasing in v for every $x > 1$, the unique solution x^* lies in $(1,\hat{x})$ — equivalently, $r^* \in (\hat{r}, 1)$ — if and only if $v \in (\underline{v}, \overline{v})$.

A.2.2 Proof for the Boston mechanism

Suppose the DA mechanism has an interior equilibrium with $r^D \in (\hat{r}, 1)$, i.e., $v \in (\underline{v}, \overline{v})$. Then r^D is the unique solution to (7) for the DA mechanism in $(\hat{r}, 1)$.

Note that $r > \hat{r}$ if and only if

$$\Delta^D(0;r) - \Delta^B(0;r) = \min\left\{\frac{\lambda_a}{1-r}, \frac{1-\lambda_a}{r}\right\} - \frac{\lambda_s}{r} > 0.$$

When $r < \hat{r}$ (or $r = \hat{r}$), the opposite strict inequality (or equality) holds. Therefore, the right-hand side of (7) under the Boston mechanism is smaller than, larger than, or equal to that under the DA mechanism when $r > \hat{r}$, $r < \hat{r}$, or $r = \hat{r}$, respectively. By the Intermediate Value Theorem, an interior equilibrium of the Boston mechanism exists with $r^B \in (\hat{r}, r^D)$.

We next prove uniqueness of the equilibrium proportion $r^B \in (\hat{r}, r^D)$ under the Boston mechanism.

Let $z \equiv \frac{\lambda_s}{\mu}$, $w \equiv \frac{\lambda_a}{\mu}$, and $y \equiv \frac{\lambda_b}{\mu}$. The equilibrium condition (7) for the Boston mechanism becomes

$$e^z = 1 + \frac{e^{\frac{z}{r}} - 1}{\frac{1-r}{r} e^{\min\{\alpha_1(r), \alpha_2(r)\}} e^{-\frac{zv}{r}} + 1} \equiv h(r), \tag{11}$$

where $\alpha_1(r) \equiv \frac{w}{1-r}$ and $\alpha_2(r) \equiv \frac{y+z}{r}$.

We search for solutions to (11) in two regions. Define $h_i(r) \equiv 1 + \frac{e^{\frac{z}{r}}-1}{\frac{1-r}{r}e^{\alpha_i(r)}e^{-\frac{zv}{r}}+1}$ for $i = 1, 2$, so that $h(r) = \max\{h_1(r), h_2(r)\}$. More precisely, if $r \leq 1 - \lambda_a$ then $\alpha_1(r) \leq \alpha_2(r)$ and $h(r) = h_1(r)$; if $r \geq 1 - \lambda_a$ then $\alpha_1(r) \geq \alpha_2(r)$ and $h(r) = h_2(r)$. We establish two claims below regarding $h_1(r)$ and $h_2(r)$.

Claim 1. *$h_1(r)$ single-crosses e^z from above to below as r increases in $[\hat{r}, 1)$. Specifically, if $e^z \geq h_1(r')$ for some $r' \in [\hat{r}, 1)$, then $e^z > h_1(r'')$ for every $r'' \in (r', 1)$.*

Proof. For any $r \in [\hat{r}, 1)$,

$$e^z \geq h_1(r) \iff \frac{1-r}{r} e^{\alpha_1(r)} e^{-\frac{zv}{r}} + 1 \geq \frac{e^{\frac{z}{r}} - 1}{e^z - 1}$$
$$\iff e^{\frac{w}{1-r}} e^{-\frac{zv}{r}} \geq \frac{e^z}{e^z - 1} \frac{(e^{z((1/r)-1)} - 1)}{(1/r) - 1}.$$

The strict inequalities hold as well. The left-hand side of the last inequality is strictly increasing in r, whereas the right-hand side is strictly decreasing in r because

$$\left(\frac{e^{z(x-1)} - 1}{x - 1} \right)' > 0 \iff z(x-1)e^{z(x-1)} > e^{z(x-1)} - 1 \iff e^{-z(x-1)} > 1 - z(x-1),$$

which holds for every $x > 1$. The claim follows. □

Claim 2. *$h_2(r)$ single-crosses e^z from above to below as r increases in $[\hat{r}, 1)$. Formally, if $e^z \geq h_2(r')$ for some $r' \in [\hat{r}, 1)$, then $e^z > h_2(r'')$ for every $r'' \in (r', 1)$.*

Proof. For any $r \in [\hat{r}, 1)$,

$$e^z \geq h_2(r) \iff 1 \geq \frac{e^{\frac{z}{r}} - 1}{e^z - 1} - \frac{1-r}{r} e^{\alpha_2(r)} e^{-\frac{zv}{r}}.$$

Substituting $x \equiv \frac{1}{r} \in (1, 1/\hat{r}]$ and $\tilde{\alpha}_2(x) \equiv (y+z)x$ yields

$$e^z \geq h_2(r) \iff 1 \geq \frac{e^{zx} - 1}{e^z - 1} - (x-1) e^{\tilde{\alpha}_2(x)} e^{-zvx} \equiv \tilde{h}_2(x). \tag{12}$$

Since $\tilde{\alpha}_2' = y + z$ is independent of x,

$$\begin{aligned} \tilde{h}_2'(x) < 0 &\iff 0 > \frac{z e^{zx}}{e^z - 1} - e^{\tilde{\alpha}_2(x) - zvx} - (x-1)(\tilde{\alpha}_2' - zv) e^{\tilde{\alpha}_2(x) - zvx} \\ &\iff 0 > \frac{z e^{z(1+v)x - \tilde{\alpha}_2(x)}}{e^z - 1} - 1 - (x-1)(\tilde{\alpha}_2' - zv) \equiv \tilde{g}(x, v). \end{aligned}$$

First, $\tilde{h}_2(1) = 1$. Second, since $v \in (\underline{v}, \overline{v})$, a unique interior equilibrium of the DA mechanism with $r^D \in (\hat{r}, 1)$ exists. The function $g(x, v)$ defined in (8) satisfies $g(1, v) < 0$, and since $\tilde{g}(1, v) < g(1; v)$, we have $\tilde{g}(1, v) < 0$, implying $\tilde{h}_2'(1) < 0$. Finally, if $zv > y$, then $\tilde{g}(x, v)$ is strictly convex in $x \in (1, 1/\hat{r}]$, so as x increases from 1 to $\frac{1}{\hat{r}}$, either $\tilde{g}(x, v)$ remains strictly negative, or its sign changes exactly once from strictly negative to strictly positive. If $zv \leq y$, then $\tilde{g}(x, v)$ is decreasing in x and remains strictly negative. In either case, $\tilde{h}_2(x)$ is strictly single-dipped in $x \in (1, 1/\hat{r}]$.

Since $\tilde{h}_2(1) = 1$, $\tilde{h}_2'(1) < 0$, and $\tilde{h}_2(x)$ is strictly single-dipped in $x \in (1, 1/\hat{r}]$, the function $\tilde{h}_2(x)$ single-crosses 1 from below to above as x increases in $(1, 1/\hat{r}]$. It follows from (12) that $h_2(r)$ single-crosses e^z from above to below as r increases in $[\hat{r}, 1)$. □

The uniqueness of the interior equilibrium fraction $r^B \in (\hat{r}, r^D)$ under the Boston mechanism follows from the following reasoning. If $h(1 - \lambda_a) \geq e^z$, then there is no solution to $h(r) = e^z$ in $(\hat{r}, 1 - \lambda_a)$ by Claim 1, and there is at most one solution in $[1 - \lambda_a, 1)$ by Claim 2. On the other hand, if $h(1 - \lambda_a) < e^z$, then there is at most one solution of $h(r) = e^z$ in $(\hat{r}, 1 - \lambda_a]$ by Claim 1, and no solution exists in $[1 - \lambda_a, 1)$ by Claim 2.

A.3 Proof of Proposition 2

For any mechanism $\Gamma \in \{\text{Boston}, \text{DA}\}$, substituting $\Delta^\Gamma(\theta; r) = \Delta^\Gamma(0; r) - \frac{\lambda_s \theta}{r}$ (from (3) and (4)) into the equilibrium condition (6) yields:

$$m^\Gamma(\theta) = \left(1 + \frac{1 - r^\Gamma}{r^\Gamma} \exp\left(-\frac{\Delta^\Gamma(0; r^\Gamma)}{\mu}\right) \exp\left(\frac{\lambda_s \theta}{r\mu}\right)\right)^{-1}.$$

The equilibrium condition (7) further implies:

$$\frac{1-r^\Gamma}{r^\Gamma}\exp\left(-\frac{\Delta^\Gamma(0;r^\Gamma)}{\mu}\right)=\frac{\exp\left(\frac{\lambda_s}{r^\Gamma\mu}\right)-1}{\exp\left(\frac{\lambda_s}{\mu}\right)-1}-1.$$

Letting $z\equiv\frac{\lambda_s}{\mu}$, we obtain:

$$m^\Gamma(\theta)=\left[1+h(\theta,r^\Gamma)\right]^{-1},\quad \text{where}\quad h(\theta,r)=\frac{\exp\left(-\frac{z\theta}{r}\right)\left(\exp\left(\frac{z}{r}\right)-\exp(z)\right)}{\exp(z)-1}.$$

Since $g^\Gamma(\theta)=m^\Gamma(\theta)\frac{\lambda_s}{r^\Gamma}$, we have:

$$\frac{d(\lambda_s/g^\Gamma(\theta))}{d\theta}=\frac{\partial(r^\Gamma(1+h(\theta,r^\Gamma))}{\partial\theta}=-z\cdot h(\theta,r^\Gamma).$$

Moreover,

$$\begin{aligned}\frac{\partial h(\theta,r)}{\partial r}&=\left[\frac{z\theta}{r^2}e^{-\frac{z\theta}{r}}(e^{\frac{z}{r}}-e^z)-\frac{z}{r^2}e^{-\frac{z\theta}{r}}e^{\frac{z}{r}}\right]\frac{1}{e^z-1}\\&=-\left[\frac{z}{r^2}e^{-\frac{z\theta}{r}}(e^{\frac{z}{r}}(1-\theta)+e^z\theta)\right]\frac{1}{e^z-1}<0.\end{aligned}$$

Therefore,

$$\begin{aligned}r^B<r^D&\implies(\forall\theta\in(0,1))\quad h(\theta,r^B)>h(\theta,r^D)\\&\implies(\forall\theta\in(0,1))\quad\frac{d(\lambda_s/g^B(\theta))}{d\theta}<\frac{d(\lambda_s/g^D(\theta))}{d\theta}\\&\implies(\forall\theta<\theta')\quad\frac{1}{g^B(\theta)}\leq\frac{1}{g^D(\theta)}\implies\frac{1}{g^B(\theta')}<\frac{1}{g^D(\theta')}\\&\implies g^B\text{ single-crosses }g^D\text{ from below.}\end{aligned}$$

A.4 Proof of Proposition 3

Expressions (3) and (4) show that both $\frac{\Delta^D(\theta;r^B)}{\mu}$ and $\frac{\Delta^B(\theta;r^B)}{\mu}$ are functions of θ of the form $\alpha(\theta+\beta)$, where $\alpha=\frac{\lambda_s}{r^B\mu}$ and $\beta=v-1$ for the DA mechanism, while for the Boston mechanism, β is smaller than $v-1$. Therefore, switching from the Boston to the DA mechanism while keeping r^B unchanged corresponds to increasing β from $\underline{\beta}=v-\min\left\{\frac{r^B}{\lambda_s}\frac{\lambda_a}{1-r^B},\frac{1-\lambda_a}{\lambda_s}\right\}$ to $\overline{\beta}=v-1$.

By (Yang, 2015), a student's interior optimal strategy $m:[0,1]\to[0,1]$, for given

α and β, satisfies the first-order condition:

$$\alpha(\theta+\beta) = \ln\left(\frac{m(\theta)}{1-m(\theta)}\right) - \ln\left(\frac{\overline{m}}{1-\overline{m}}\right),$$

where $\overline{m}$ denotes the expected value of $m(\theta)$ over the uniform distribution of θ. Thus, an optimal strategy takes the form $m(\theta) = \frac{Le^{\alpha(\theta+\beta)}}{Le^{\alpha(\theta+\beta)}+1}$, where $L = \frac{\overline{m}}{1-\overline{m}}$ is the likelihood ratio between reports *sab* and *asb*. The consistency condition $\overline{m} = \int m(\theta)\, d\theta$ requires L to satisfy

$$\int_0^1 \frac{Le^{\alpha(\theta+\beta)}}{Le^{\alpha(\theta+\beta)}+1}\, d\theta = \frac{L}{L+1} \iff \int_0^1 \frac{1}{L+e^{-\alpha(\theta+\beta)}}\, d\theta = \frac{1}{L+1}. \tag{13}$$

We first establish the existence of a unique interior solution L to (13). Finding a solution L to (13) is equivalent to finding $\overline{m} = \frac{L}{L+1} \in (0,1)$ satisfying:

$$\begin{aligned}
&\int_0^1 \frac{\overline{m}e^{\alpha(\theta+\beta)}}{\overline{m}e^{\alpha(\theta+\beta)}+1-\overline{m}}\, d\theta - \overline{m} = 0 \\
\iff\ & \log(\overline{m}e^{\alpha(\theta+\beta)}+1-\overline{m})\Big|_0^1 - \alpha\overline{m} = 0 \\
\iff\ & f(\overline{m};\alpha,\beta) \equiv \overline{m}(e^{\alpha(1+\beta)} - e^{\alpha(\overline{m}+\beta)}) + (1-\overline{m})(1-e^{\alpha\overline{m}}) = 0. \qquad (14)
\end{aligned}$$

Since $v \in (\underline{v}, \overline{v})$, both the Boston and DA mechanisms have unique interior equilibria, with the Boston equilibrium proportion $r^B \in (\hat{r}, 1)$ satisfying $f(r^B;\alpha,\underline{\beta}) = 0$.

For any $\beta^* \in (\underline{\beta}, \overline{\beta}]$, we have $0 = f(r^B;\alpha,\underline{\beta}) < f(r^B;\alpha,\beta^*)$, since f is increasing in β. Moreover, $f(1;\alpha,\beta^*) = 0$, and since $1+\beta^* \leq 1+\overline{\beta} = v < \overline{v}$,

$$f'(1;\alpha,\beta^*) = e^{\alpha} - 1 - \alpha e^{\alpha(1+\beta^*)} > e^z - 1 - ze^{z\overline{v}} = 0 \quad \text{(by (9))}.$$

Therefore, there exists $\overline{m} \in (r^B, 1)$ such that $f(\overline{m};\alpha,\beta^*) = 0$, which is equivalent to $L^* = \frac{\overline{m}}{1-\overline{m}}$ being a solution to (13) at $\beta^* > \underline{\beta}$.

The solution L^* to (13) at $\beta^* > \underline{\beta}$ is unique because

$$\frac{d}{dL}\left(\int_0^1 \frac{1}{e^{-\alpha(\theta+\beta^*)}+L^*}\, d\theta - \frac{1}{L^*+1}\right) = -\int_0^1 \frac{1}{(e^{-\alpha(\theta+\beta^*)}+L^*)^2}\, d\theta + \frac{1}{(L^*+1)^2} < 0.$$

The inequality holds because $\frac{1}{e^{-\alpha(\theta+\beta^*)}+L^*}$ is a random variable with expected value $\frac{1}{L^*+1}$ by (13), and $\text{Var}[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 > 0$ for any random variable X.

Let $L(\beta)$ denote the unique solution to (13) for each $\beta \in (\underline{\beta}, \overline{\beta})$. By the Implicit

Function Theorem,

$$\int_0^1 \frac{-\alpha e^{-\alpha(\theta+\beta)} + L'(\beta)}{(e^{-\alpha(\theta+\beta)} + L(\beta))^2}\, d\theta = \frac{L'(\beta)}{(L(\beta)+1)^2},$$

which implies

$$L'(\beta)\left[\int_0^1 \frac{1}{(e^{-\alpha(\theta+\beta)} + L(\beta))^2}\, d\theta - \frac{1}{(L(\beta)+1)^2}\right] = \int_0^1 \frac{\alpha e^{-\alpha(\theta+\beta)}}{(e^{-\alpha(\theta+\beta)} + L(\beta))^2}\, d\theta > 0.$$

Therefore, $L'(\beta) > 0$.

Finally, take any $\beta_1 < \beta_2$ in $(\underline{\beta}, \overline{\beta})$, and let L_1 and L_2 be the solutions to (13) for β_1 and β_2, respectively. Since $L'(\beta) > 0$, we have $L_1 < L_2$. For each $i \in \{1, 2\}$, let $m_i(\theta)$ be the optimal strategy and $g_i(\theta) \equiv m_i(\theta)\frac{\lambda_s}{\overline{m}_i}$ the resulting assignment of students to school s under the DA mechanism. Then, $\frac{\lambda_s}{g_i(\theta)} = \frac{L_i + e^{-\alpha(\theta+\beta_i)}}{L_i+1}$, and

$$\frac{\lambda_s}{g_2(\theta)} - \frac{\lambda_s}{g_1(\theta)} = \left(\frac{e^{-\alpha\beta_2}}{L_2+1} - \frac{e^{-\alpha\beta_1}}{L_1+1}\right) e^{-\alpha\theta} + \left(\frac{L_2}{L_2+1} - \frac{L_1}{L_1+1}\right),$$

which is strictly increasing in θ. Hence, g_1 single-crosses g_2 from below.

A.5 Proof of Propositions 4 and 5

Propositions 4 and 5 both analyze the student's problem under the DA mechanism, but with different assumptions: Proposition 4 varies the proportion r of other students reporting *sab* while holding μ fixed, whereas Proposition 5 varies μ while holding r fixed.

For any μ and r such that $\mu < \lambda_s(1-v)$ and $r \in (r_\mu^B, r_\mu^D)$, we have $\frac{\Delta^D(\theta;r)}{\mu} = \alpha(\theta + v - 1)$, where $\alpha = \frac{\lambda_s}{r\mu}$. A decrease in α corresponds to either an increase in r with μ fixed (for Proposition 4) or an increase in μ with r fixed (for Proposition 5).

In the proof of Proposition 3, we showed that a student's interior optimal strategy is $m(\theta) = \frac{\overline{m}e^{\alpha(\theta+\beta)}}{\overline{m}e^{\alpha(\theta+\beta)}+1-\overline{m}}$, where $\overline{m}$ is the unique interior solution to (14). Setting $\beta = v-1$ for the DA mechanism, this is equivalently written as

$$h(\overline{m}, \alpha) \equiv e^{-\alpha\overline{m}} f(\overline{m}; \alpha, v-1) = \overline{m}(e^{-\alpha(\overline{m}-v)} - e^{-\alpha(1-v)}) + (1-\overline{m})(e^{-\alpha\overline{m}} - 1) = 0.$$

Note that α varies while $\beta = v - 1$ is held fixed, since we consider only the DA mechanism.

Part 1. We show that the unique solution $\overline{m}^*$ to $h(\overline{m}, \alpha) = 0$ in $(\hat{r}, 1)$ decreases in

α. This implies that the homogeneity of rank-order reports increases in the belief r (Proposition 4) and in the information cost μ (Proposition 5).

Note that $h(0,\alpha) = h(1,\alpha) = 0$. Moreover,

$$\frac{\partial h(0,\alpha)}{\partial \overline{m}} = e^{\alpha v}(1-e^{-\alpha}) - \alpha \geq e^{\alpha/2} - e^{-\alpha/2} - \alpha > 0, \quad \text{and}$$
$$\frac{\partial h(1,\alpha)}{\partial \overline{m}} = e^{-\alpha}(e^{\alpha} - 1 - \alpha e^{\alpha v}) > e^{-\alpha}(e^{\alpha} - 1 - \alpha e^{\alpha \overline{v}}) > 0,$$

where the last inequality holds because $e^{\alpha} - 1 - \alpha e^{\alpha\overline{v}}$ increases in α, $e^{z} - 1 - ze^{z\overline{v}} = 0$ by (9), and $\alpha \equiv \frac{\lambda_s}{r\mu} > \frac{\lambda_s}{\mu} \equiv z$. Together, these inequalities imply that at the unique interior solution $\overline{m}^* \in (0,1)$ to $h(\overline{m},\alpha) = 0$, we must have $\frac{\partial h(\overline{m}^*,\alpha)}{\partial \overline{m}} < 0$.

Since $\mu < \lambda_s(1-v)$, we have $\frac{1}{\alpha}+v < \frac{\mu}{\lambda_s}+v < 1$, which implies that at $\overline{m} = \frac{1}{\alpha}+v < 1$,

$$h(\overline{m},\alpha) = \frac{1}{\alpha e}\left((1+\alpha v)(1-e^{1-\alpha(1-v)}) + (\alpha(1-v)-1)(e^{\alpha v} - e)\right) > 0,$$

and therefore $\overline{m}^* > \frac{1}{\alpha} + v$. It follows that $1 < \alpha(\overline{m}^* - v) < \alpha(1-v)$, and

$$\begin{aligned}\frac{\partial h(\overline{m}^*,\alpha)}{\partial \alpha} &< \overline{m}^* \frac{\partial}{\partial \alpha}\left(e^{-\alpha(\overline{m}^*-v)} - e^{-\alpha(1-v)}\right) \\ &= \frac{1}{\alpha}\left(-\alpha(\overline{m}^*-v)e^{-\alpha(\overline{m}^*-v)} + \alpha(1-v)e^{-\alpha(1-v)}\right) < 0,\end{aligned}$$

where the last inequality holds because xe^{-x} is decreasing in $x > 1$. By the Implicit Function Theorem, $\overline{m}^*$ decreases in α, or equivalently, increases in r and μ.

Part 2. Let $L(\alpha)$ denote the unique solution to (14). Then $m(\theta;\alpha) = \frac{L(\alpha)e^{\alpha(\theta+v-1)}}{L(\alpha)e^{\alpha(\theta+v-1)}+1}$ and

$$\frac{g(\theta;\alpha)}{\lambda_s} = \frac{L(\alpha)+1}{L(\alpha)+e^{-\alpha(\theta+v-1)}}.$$

In Part 1, we showed that $L(\alpha) = \frac{\overline{m}(\alpha)}{1-\overline{m}(\alpha)}$ decreases in α. Moreover,

$$\begin{aligned}\frac{\partial g(\theta;\alpha)}{\partial \alpha} > 0 &\iff L'(\alpha)(L(\alpha)+e^{-\alpha(\theta+v-1)}) > (L(\alpha)+1)(L'(\alpha) - (\theta+v-1)e^{-\alpha(\theta+v-1)}) \\ &\iff \theta + v - 1 > \frac{L'(\alpha)}{L(\alpha)+1}(e^{\alpha(\theta+v-1)} - 1).\end{aligned}$$

Since $L'(\alpha) < 0$ and $e^{\alpha(\theta+v-1)} - 1 < 0$ for $\theta < 1-v$, the right-hand side is positive for $\theta < 1-v$ and negative for $\theta > 1-v$. Therefore, $\frac{\partial g(\theta;\alpha)}{\partial \alpha} > 0$ for every $\theta > 1-v$ and $\frac{\partial g(\theta;\alpha)}{\partial \alpha} < 0$ for every $\theta < 1-v$. It follows that $g^H(\cdot;\alpha_1)$ single-crosses $g^H(\cdot;\alpha_2)$ from below whenever $\alpha_1 > \alpha_2$.

A.6 Homogeneity of Rank-Order Reports in μ

Proposition 6. *Assume $v > 1/2$ and $\lambda_s \leq \lambda_a$, which implies that school s is more competitive than school a under any mechanism and for any $\mu \geq 0$.*[11]

1. *There exists $\overline{\mu} > 0$ such that, as μ increases from 0 to $\overline{\mu}$, the equilibrium fraction r^D under the DA mechanism increases from v to 1.*
2. *Let r^B_μ and $r^B_{\mu'}$ denote the equilibrium fractions under the Boston mechanism for $\mu < \mu'$, respectively. If $r^B_\mu > \frac{1}{2} \geq \frac{\lambda_s}{\lambda_s+\lambda_a}$, then $r^B_\mu < r^B_{\mu'}$. Moreover, if $v \leq \frac{1}{2} + \frac{\lambda_b}{\lambda_s}$, then the equilibrium fraction r^B_μ is bounded above by $\max\left\{\frac{1}{2}, 1-\lambda_a\right\}$ for any μ.*

A.6.1 Proof of Proposition 6 for the DA mechanism

For a given μ, let $\overline{v}_\mu$ denote the upper bound of v for which an interior equilibrium with $r^D_\mu \in (\hat{r}, 1)$ exists. Specifically, $\overline{v}_\mu = \frac{1}{z}\log\left(\frac{e^z-1}{z}\right)$, where $z \equiv \frac{\lambda_s}{\mu}$, as defined in (9). One can show that $\overline{v}_\mu$ is strictly increasing and continuous in z, ranging from $\lim_{z\to 0}\overline{v}_\mu = \frac{1}{2}$ to $\lim_{z\to\infty}\overline{v}_\mu = 1$. Consequently, for each $v > \frac{1}{2}$, there exists $\overline{\mu}$ such that $v = \frac{1}{\underline{z}}\log\left(\frac{e^{\underline{z}}-1}{\underline{z}}\right)$, where $\underline{z} \equiv \frac{\lambda_s}{\overline{\mu}}$. An interior equilibrium under the DA mechanism with $r^D_\mu \in (\hat{r}, 1)$ exists if and only if $\mu < \overline{\mu}$.

Moreover, $\lim_{\mu\to\overline{\mu}}\overline{v}_\mu = v$, meaning the upper bound for an interior equilibrium approaches v, and therefore $\lim_{\mu\to\overline{\mu}} r^D_\mu = 1$.

We now show that the equilibrium fraction r^D_μ is strictly increasing in μ.

Let $x \equiv \frac{1}{r}$ and $z \equiv \frac{\lambda_s}{\mu}$. The equilibrium condition (7) can be written as:

$$
\begin{aligned}
e^z &= 1 + \frac{e^{zx}-1}{(x-1)e^{zx(1-v)}+1} \\
&\iff e^{zx} - 1 - (e^z-1)((x-1)e^{zx(1-v)}+1) = 0 \\
&\iff (e^{zx}-e^z) - (x-1)(e^z-1)e^{zx(1-v)} = 0 \\
&\iff h(x;z) \equiv e^{zxv}(1-e^{z(1-x)}) - (x-1)(e^z-1) = 0.
\end{aligned}
$$

By Proposition 1, there exists a unique solution x^* to $h(x;z) = 0$. To apply the Implicit Function Theorem, we establish the following claims. For ease of exposition, we suppress the dependence of x^* on z.

Claim 3. *1. $\frac{1}{y} - \frac{1}{e^y-1} < \frac{1}{2}$ for any $y > 0$, and*

[11] This follows from observing that school s is more competitive both in expected match payoffs, i.e., $E[u_s] = v + \frac{1}{2} > u_a = 1$, and in ex-post preferences, i.e., $\Pr[u_s > u_a] = v$, relative to capacities, since $v > \frac{\lambda_s}{\lambda_s+\lambda_a}$.

2. $\log\left(\frac{y}{e^y-1}\right)+y+\frac{y}{e^y-1}$ *is strictly increasing in* y.

Proof. **Part 1.** By L'Hôpital's rule,

$$\lim_{y\to 0}\frac{1}{y}-\frac{1}{e^y-1}=\lim_{y\to 0}\frac{e^y-1-y}{y(e^y-1)}=\lim_{y\to 0}\frac{e^y-1}{e^y-1+ye^y}=\lim_{y\to 0}\frac{e^y}{2e^y+ye^y}=\frac{1}{2}.$$

Moreover,

$$\begin{aligned}\left(\frac{1}{y}-\frac{1}{e^y-1}\right)'&=-\frac{1}{y^2}+\frac{e^y}{(e^y-1)^2}<0\iff y^2e^y<(e^y-1)^2 \qquad (15)\\ &\Longleftarrow 2ye^y+y^2e^y<2(e^y-1)e^y \iff 2y+y^2<2(e^y-1)\\ &\Longleftarrow 2+2y<2e^y,\end{aligned}$$

which holds for every $y>0$. In each step, we verify that the two sides converge as $y\to 0$ and compare their derivatives.

Part 2. Let $f(y)\equiv\log\left(\frac{y}{e^y-1}\right)+y+\frac{y}{e^y-1}$. Then,

$$\begin{aligned}f'(y)=1+\left(1+\frac{e^y-1}{y}\right)\left(\frac{y}{e^y-1}\right)'&=1+\left(1+\frac{e^y-1}{y}\right)\frac{e^y-1-ye^y}{(e^y-1)^2}>0\\ \iff \left(1+\frac{e^y-1}{y}\right)\left(1-\frac{ye^y}{e^y-1}\right)&>1-e^y\\ \iff \frac{e^y-1}{y}&>\frac{ye^y}{e^y-1},\end{aligned}$$

which holds by (15). □

Claim 4. $\frac{\partial h(x^*,z)}{\partial x}>0$.

Proof. Observe that

$$\frac{\partial h(x,z)}{\partial x}=e^{zxv}(zv)(1-e^{z(1-x)})+e^{zxv}e^{z(1-x)}z-(e^z-1).$$

Since $h(x^*,z)=0$, we have $\frac{e^{zx^*v}(1-e^{z(1-x^*)})}{x^*-1}=e^z-1$, which implies

$$\begin{aligned}\frac{\partial h(x^*,z)}{\partial x}>0 &\iff e^{zx^*v}(zv)(1-e^{z(1-x^*)})+e^{zx^*v}e^{z(1-x^*)}z>(e^z-1)\\ &\iff zv(x^*-1)+\frac{z(x^*-1)}{1-e^{z(1-x^*)}}e^{z(1-x^*)}>1\\ &\iff v>\frac{1}{z(x^*-1)}-\frac{1}{e^{z(x^*-1)}-1}.\end{aligned}$$

The last inequality holds by $v>1/2$ and Part 1 of Claim 3. □

Claim 5. *If $x^* < 2$, then $\frac{\partial h(x^*,z)}{\partial z} < 0$.*

Proof. Observe that

$$\frac{\partial h(x,z)}{\partial z} = e^{zxv}(xv)(1 - e^{z(1-x)}) + e^{zxv}e^{z(1-x)}(x-1) - (x-1)e^z.$$

Since $h(x^*, z) = 0$, we have $e^{zx^*v} = \frac{(x^*-1)(e^z-1)}{1-e^{z(1-x^*)}}$, which implies

$$\begin{aligned}\frac{\partial h(x^*,z)}{\partial z} < 0 &\iff (x^*v) + \frac{(x^*-1)e^{z(1-x^*)}}{1-e^{z(1-x^*)}} < \frac{e^z}{e^z-1} \\ &\iff (x^*v) + \frac{(x^*-1)}{e^{z(x^*-1)}-1} < 1 + \frac{1}{e^z-1}.\end{aligned}$$

On the other hand, $h(x^*, z) = 0$ is equivalent to

$$\begin{aligned}zx^*v &= \log\left(\frac{(x^*-1)(e^z-1)}{1-e^{z(1-x^*)}}\right) = \log\left(\frac{z(x^*-1)}{1-e^{z(1-x^*)}}\right) + \log\left(\frac{e^z-1}{z}\right) \\ &= \log\left(\frac{z(x^*-1)}{e^{z(x^*-1)-1}}\right) + z(x^*-1) - \log\left(\frac{z}{e^z-1}\right).\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{\partial h(x^*,z)}{\partial z} < 0 &\iff (zx^*v) + \frac{z(x^*-1)}{e^{z(x^*-1)}-1} < z + \frac{z}{e^z-1} \\ &\iff \log\left(\frac{z(x^*-1)}{e^{z(x^*-1)-1}}\right) + z(x^*-1) + \frac{z(x^*-1)}{e^{z(x^*-1)}-1} < \log\left(\frac{z}{e^z-1}\right) + z + \frac{z}{e^z-1}.\end{aligned}$$

The last inequality holds for $x^* < 2$ by Part 2 of Claim 3. □

When μ is sufficiently small (i.e., z is large), r^D_μ is close to the equilibrium fraction $v > \frac{1}{2}$ under zero information cost, so $r^D_\mu > 1/2$ (i.e., $x^* < 2$). When $x^* < 2$, it follows from Claim 4, Claim 5, and the Implicit Function Theorem that x^* increases in z (i.e., decreases in μ). Hence, as μ increases (i.e., z decreases), r^D_μ remains greater than 1/2 (i.e., x^* remains less than 2) and continues to increase.

A.6.2 Proof of Proposition 6 for the Boston mechanism

Assume v and μ are such that an equilibrium fraction r^B_μ exists in $(1/2, 1)$. Using the change of variables $x \equiv \frac{1}{r}$, $z \equiv \frac{\lambda_s}{\mu}$, $w \equiv \frac{\lambda_a}{\mu}$, and $y \equiv \frac{\lambda_b}{\mu}$, the equilibrium condition (11) becomes

$$e^z = 1 + \frac{e^{zx} - 1}{(x-1)e^{\min\{\alpha_1(x),\alpha_2(x)\}}e^{-zvx} + 1}, \tag{16}$$

where $\alpha_1(x) \equiv \frac{wx}{x-1}$ and $\alpha_2(x) \equiv (y+z)x$. For $i = 1, 2$, define

$$h_i(x, \mu) \equiv e^{zvx}(1 - e^{z(1-x)}) - (x-1)(e^z - 1)e^{\alpha_i(x) - zx}.$$

Note that if $x^B_\mu \geq \frac{1}{1-\lambda_a}$ then $h_1(x^B_\mu, \mu) = 0$, and if $x^B_\mu \leq \frac{1}{1-\lambda_a}$ then $h_2(x^B_\mu, \mu) = 0$.

We show that for $i = 1, 2$, if a solution x^*_i to $h_i(x; \mu) = 0$ is less than 2, then it strictly decreases as μ increases. The argument follows the same approach as the proof for the DA mechanism, applying the Implicit Function Theorem to show that r^D_μ increases in μ. To do so, we establish the following claim.

Claim 6. *For any $x \in (1, 2)$ and $y > 0$, $\log \frac{(x-1)(e^z-1)}{1-e^{z(1-x)}} > \frac{zx}{2}$.*

Proof. It suffices to show that for any $x \in (1, 2)$,

$$(x-1)(e^y - 1) > e^{xy/2} - e^{y(1-(x/2))}.$$

Both sides equal 0 at $x = 1$ and $e^y - 1$ at $x = 2$. The left-hand side is linear in x, while the right-hand side is strictly convex in $x \in (1, 2)$, since its second derivative is

$$(e^{xy/2} - e^{y(1-(x/2))})'' = (y/2)(e^{xy/2} + e^{y(1-(x/2))})' = (y/2)^2(e^{xy/2} - e^{y(1-(x/2))}) > 0.$$

□

Claim 7. *For $i = 1, 2$, if $x^*_i \in (1, 2)$, then $\frac{\partial h_i(x^*_i, \mu)}{\partial x} > 0$.*

Proof. For each $i = 1, 2$,

$$\begin{aligned}\frac{\partial h_i(x, \mu)}{\partial x} =& e^{zvx} zv(1 - e^{z(1-x)}) + e^{zvx}e^{z(1-x)}z \\ & - (e^z - 1)e^{\alpha_i - zx} - (x-1)(e^z - 1)e^{\alpha_i - zx}(\alpha'_i - z),\end{aligned}$$

where $\alpha'_1 \equiv \frac{d\alpha_1}{dx} = -\frac{w}{(x-1)^2}$ and $\alpha'_2 \equiv \frac{d\alpha_2}{dx} = y + z$. Since $h_i(x^*_i, \mu) = 0$, we have $e^{zvx^*_i}(1 - e^{z(1-x^*_i)}) = (x^*_i - 1)(e^z - 1)e^{\alpha_i - zx^*_i}$. Dividing by $(x^*_i - 1)(e^z - 1)e^{\alpha_i - zx^*_i}/z$ yields:

$$\begin{aligned}\frac{\partial h_i(x^*_i, \mu)}{\partial x} > 0 &\iff v + \frac{e^{z(1-x^*_i)}}{1 - e^{z(1-x^*_i)}} > \frac{1}{z(x^*_i - 1)} + \left(\frac{\alpha'_i}{z} - 1\right) \\ &\iff v - \left(\frac{\alpha'_i}{z} - 1\right) > \frac{1}{z(x^*_i - 1)} - \frac{1}{e^{z(x^*_i - 1)} - 1}. \qquad (17)\end{aligned}$$

The right-hand side of (17) is less than $\frac{1}{2}$ by Part 1 of Claim 3. The left-hand side exceeds $\frac{1}{2}$ because:

- For $i = 1$: $v - \left(\frac{\alpha_1'}{z} - 1\right) > v + 1 > \frac{1}{2}$.
- For $i = 2$: $h_2(x_2^*, \mu) = 0$ implies $zvx_2^* = \log \frac{(x_2^*-1)(e^z-1)}{1-e^{z(1-x_2^*)}} + (\alpha_2 - zx_2^*)$, and since $\alpha_2 = \alpha_2' x_2^*$,

$$v - \left(\frac{\alpha_2'}{z} - 1\right) = v + 1 - \frac{\alpha_2}{zx_2^*} > \frac{1}{2} \iff \log \frac{(x_2^* - 1)(e^z - 1)}{1 - e^{z(1-x_2^*)}} > \frac{zx_2^*}{2},$$

which holds by Claim 6.

□

Claim 8. *For $i = 1, 2$, if $x_i^* \in (1, 2)$, then $\frac{\partial h_i(x_i^*,\mu)}{\partial \mu} > 0$.*

Proof. For each $i = 1, 2$,

$$\begin{aligned}\frac{\partial h_i(x, \mu)}{\partial \mu} =& z'\left(e^{zvx} vx(1 - e^{z(1-x)}) - e^{zvx} e^{z(1-x)}(1 - x) - (x - 1)e^z e^{\alpha_i - zx}\right) \\ & - (x - 1)(e^z - 1)e^{\alpha_i - zx}(\alpha_i' - z'x),\end{aligned}$$

where $z' = -\frac{\lambda_s}{\mu^2}$, $w' = -\frac{\lambda_a}{\mu^2}$, $y' = -\frac{\lambda_b}{\mu^2}$, $\alpha_1' = \frac{w'x}{x-1} < 0$, and $\alpha_2' = (y' + z')x < 0$. Since $h_i(x_i^*, \mu) = 0$, we have $e^{zvx_i^*}(1 - e^{z(1-x_i^*)}) = (x_i^* - 1)(e^z - 1)e^{\alpha_i - zx_i^*}$. Dividing by $(x_i^* - 1)(e^z - 1)e^{\alpha_i - zx_i^*}/z'$ yields:

$$\begin{aligned}\frac{\partial h_i(x_i^*, \mu)}{\partial \mu} < 0 &\iff vx_i^* - \frac{e^{z(1-x_i^*)}(1 - x_i^*)}{1 - e^{z(1-x_i^*)}} < \frac{e^z}{e^z - 1} + \left(\frac{\alpha_i'}{z'} - x_i^*\right) \\ &\iff zvx_i^* + \frac{z(x_i^* - 1)}{e^{z(x_i^*-1)} - 1} < z + \frac{z}{e^z - 1} + (\alpha_i - zx_i^*),\end{aligned}$$

where the last step uses $\frac{\alpha_i'}{z'} = \frac{\alpha_i}{z}$ for $i = 1, 2$. On the other hand, $h_i(x_i^*, \mu) = 0$ implies

$$\begin{aligned}zvx_i^* &= \log\left(\frac{(x_i^* - 1)(e^z - 1)e^{\alpha_i - zx_i^*}}{1 - e^{z(1-x_i^*)}}\right) \\ &= \log\left(\frac{z(x_i^* - 1)}{e^{z(x_i^*-1)} - 1}\right) + z(x_i^* - 1) - \log\left(\frac{z}{e^z - 1}\right) + (\alpha_i - zx_i^*).\end{aligned}$$

Therefore, $\frac{\partial h_i(x_i^*,\mu)}{\partial \mu} < 0$ if and only if

$$\log\left(\frac{z(x_i^* - 1)}{e^{z(x_i^*-1)} - 1}\right) + z(x_i^* - 1) + \frac{z(x_i^* - 1)}{e^{z(x_i^*-1)} - 1} < \log\left(\frac{z}{e^z - 1}\right) + z + \frac{z}{e^z - 1},$$

which holds by $x_i^* < 2$ and Part 2 of Claim 3. □

For each $i = 1, 2$, if the solution x_i^* to $h_i(x, \mu) = 0$ is less than 2, then it decreases as μ increases by Claim 7, Claim 8, and the Implicit Function Theorem. Since $x_\mu^B = x_i^*$ for either $i = 1$ or $i = 2$, it follows that $x_\mu^B < 2$ also decreases as μ increases. Consequently, $r_\mu^B = 1/x_\mu^B$ increases as μ increases whenever $r_\mu^B > 1/2$.

We next show that $v \leq \frac{1}{2} + \frac{\lambda_b}{\lambda_s}$ implies $r_\mu^B \leq \max\left\{\frac{1}{2}, 1 - \lambda_a\right\}$. Suppose $r_\mu^B > 1 - \lambda_a$. Then by the equilibrium condition (11), r_μ^B is the unique solution to

$$e^z = 1 + \frac{e^{\frac{z}{r}} - 1}{\frac{1-r}{r} e^{\frac{y+z}{r}} e^{-\frac{zv}{r}} + 1} \equiv \tilde{h}(r).$$

By Claim 2, $\tilde{h}(r)$ single-crosses e^z from above to below as r increases in $[\hat{r}, 1)$. Moreover,

$$\begin{aligned} e^z \geq \tilde{h}(1/2) &\iff e^{2(y+z)} e^{-2zv} + 1 \geq \frac{e^{2z} - 1}{e^z - 1} = e^z + 1 \\ &\iff 2(y + z - zv) > z \iff v \leq \frac{1}{2} + \frac{y}{z} = \frac{1}{2} + \frac{\lambda_b}{\lambda_s}. \end{aligned}$$

Therefore, $r_\mu^B \leq 1/2$.

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