

# An Experiment on Behavior in Queues

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## Abstract

In a dynamic allocation setup, we experimentally study agents' choices under the First-In-First-Out (FIFO) and Last-In-First-Out (LIFO) queuing protocols. We find that agents are nearly rational under FIFO but tend to be overly selective under LIFO. In the model that anchors our experiment, we show that the magnitude of such an excessively selective bias reduces the welfare performance gap between FIFO and LIFO. Since strategic complementarities can reinforce overly selective behavior, we show that such bias persists in a supplemental LIFO treatment where subjects interact with non-strategic robots.

**Keywords:** Dynamic Matching, Priority Protocols, Queuing.

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# 1 Introduction

We study a dynamic allocation setting where agents match with items over time. Examples of these environments are widespread, including the assignment of public housing, medical appointments, deceased donors’ organs, etc. The theoretical literature has compared alternative queuing protocols, assessing their relative efficiency performances. Specifically, much attention has been given to the First-In-First-Out (FIFO) and the Last-In-First-Out (LIFO) protocols. The FIFO protocol is the most common, as priority is often tied to arrival times. For example, a first-come-first-served rule is used by restaurants to allocate diners to tables, or by universities to assign course slots to students. Although less common than FIFO, LIFO arises in some applications.<sup>1</sup> For example, in child adoption, adoptive parents tend to prefer younger children (see Baccara et al. (2014)). Then, birth mothers of later-born children have priority in the choice of adoptive parents, resulting in a setting equivalent to a LIFO protocol.<sup>2</sup>

Since an agent’s priority deteriorates over time under LIFO, but improves under FIFO, agents are expected to be less selective under LIFO than under FIFO. This difference in queuing behavior implies mixed efficiency comparisons depending on the theoretical model under consideration. While a more selective behavior under FIFO yields the benefits of increased market thickness, it also imposes well-known negative queuing externalities when waiting costs are present, and could result in the LIFO protocol being socially more desirable.

The relative performances of these protocols in real life also depend on how well the individuals’ behavior approximates the canonical rationality assumption underlying the theoretical models. To explore this subject, in this paper, we design an experiment to study agents’ behavior in FIFO and LIFO queues. Exploiting the model that anchors the experiment, we also evaluate the efficiency implications of the behavioral patterns observed in the experiment.

Specifically, we build a dynamic allocation model with no waiting costs where, at every period, one agent and one item arrive at the market. Agents have a lifespan of two periods, so any agent on the market is either “young” or “old.” Items’ values are common across agents and drawn independently from the same uniform distribution. An item can be offered to an agent only if it is *compatible* with her, and compatibility is independent across items and

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<sup>1</sup>Implementing LIFO is challenging, since it is subject to manipulation, and considered “unfair” as individuals who just arrived are served first (see Kahneman et al. (1986), Margaria (2024), and references therein).

<sup>2</sup>Similar considerations apply to perishable goods and to environments where the desirability of a match decreases in the time spent on the market (e.g., unemployed workers, or houses on the real estate market).

agents. At any period, an arriving item is offered to agents according to either a FIFO or a LIFO protocol, conditional on compatibility. If the item is accepted by an agent, the agent and the item leave the market matched. If the item remains unmatched, it is discarded at the end of the period. Any agent at the end of her lifespan leaves the market unmatched.

In a rational equilibrium, a young agent accepts an item if and only if its value exceeds the agent's continuation value from waiting. We show that, under FIFO, the rational equilibrium threshold is higher than the socially optimal one, while under LIFO, it is lower. In our setting, the rational equilibrium under FIFO is more efficient than under LIFO.

Next, we consider "behavioral equilibria," in which a young agent still follows a threshold strategy, but the threshold is allowed to be different from her continuation value. We only require behavioral equilibria to satisfy *Minimal Queuing Rationality* (or *MQ-rationality*). Specifically, a MQ-rational agent understands that her prospects in the next period (i) are worse than being offered the next item for sure, and (ii) deteriorate more under LIFO than under FIFO. We show that if agents behave rationally under FIFO, the efficiency gap between the FIFO and LIFO protocols first decreases and then increases again as agents become increasingly overselective under LIFO (with respect to the rational benchmark).

We design a laboratory experiment featuring reduced-form settings that are strategically equivalent to the FIFO and LIFO protocols. We find that the subjects satisfy MQ-rationality, and their behavior under FIFO converges quickly to the rational benchmark. However, under LIFO, the subjects remain consistently overselective with respect to the rational benchmark. The magnitude of this bias reduces the efficiency gap between FIFO and LIFO, bringing the efficiency under LIFO closer to the efficiency generated by the (approximately rational) behavior under FIFO.

The observation that subjects under LIFO are overselective with respect to the rational benchmark has implications broader than our setting. Specifically, introducing substantial waiting costs into our model reverses the protocols' efficiency ranking because, as mentioned above, queuing externalities arise under FIFO. Since the optimal threshold under LIFO is higher than the rational equilibrium one, a moderate overselective bias under LIFO would still improve the protocol's performance, increasing the welfare gap between FIFO and LIFO. More generally, the welfare implications of our findings vary depending on the model at hand.

The LIFO setup displays strategic complementarities, where, if a subject expects others to be overselective, the best response is to be overselective herself. To quantify this effect, in a supplemental treatment, subjects play the reduced-form LIFO game against a non-strategic

robot. We find that overselectivity persists in this setting, though to a lesser extent.<sup>3</sup>

**Related Literature** Dynamic allocation models, in which agents obtain items over time, have received significant attention in the literature (for a survey, see Baccara and Yariv (2021)).

Many papers have explored the trade-offs associated with alternative priority protocols. As mentioned above, with waiting costs, agents under FIFO impose negative externalities on others if they decide to stay in the queue rather than leave. Since LIFO rules out such externalities, it often achieves socially superior outcomes (see Naor (1969); Hassin (1985); Hassin and Haviv (2003); Su and Zenios (2004); Baccara et al. (2020)).

Generally, welfare rankings across protocols vary depending on the specific model under consideration. Notably, Bloch and Cantala (2017) study a dynamic allocation model with a fixed-length queue, where FIFO can be optimal among protocols assigning some priority to agents who arrived first (i.e., mixtures between FIFO and uniform random priority, hence excluding LIFO). The negative externality mentioned above does not arise in their setup because each agent’s decision does not affect the queue length.

The experimental and empirical literature on queuing behavior is sparse, and to our knowledge, it has not addressed behavioral patterns across protocols. Conte et al. (2014) considers an experiment where agents under time pressure select a FIFO queue among multiple ones differing in length, server speed, and entry fee. Dold and Khadjavi (2017) estimates the willingness to pay for a more favorable queue slot under FIFO. Kremer and Debo (2012) experimentally study queue-joining behavior when service quality is uncertain and, therefore, herding may arise. On the empirical side, Batt and Terwiesch (2015) and Chan et al. (2017) document the impact of delays on queue abandonment and outcomes in medical emergency departments.

## 2 The Model

### 2.1 Setup

We consider an infinite-horizon matching market between agents and items. At the beginning of each period,  $t \in \{1, 2, \dots\}$ , one agent and one item arrive on the market. Although each item must leave at the end of its arrival period, an agent can stay on the market for up to two periods before leaving. We refer to agents in their first and second periods on the market

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<sup>3</sup>In the Supplemental Appendix, we employ well-known behavioral models to explain our findings.

as “young” and “old,” respectively. When two agents are present simultaneously, they are ranked according to their arrival times by a first-in-first-out (FIFO) or a last-in-first-out (LIFO) protocol.

Upon arrival, each item’s value  $\theta$  is drawn independently from the uniform distribution over  $[0, 1]$ , and it becomes publicly known. Any agent matched with the item receives the payoff  $\theta$ . When an agent leaves the market unmatched – that is, without having obtained any item – she receives a zero payoff. In addition, an item is compatible with an agent with probability  $p \in (0, 1)$ , independently across items and agents. An agent can be offered an item only if the item is compatible with her. There is no discounting or waiting cost to stay on the market.

Once an item of value  $\theta$  enters the market, the first-ranked agent chooses whether to accept the item or not, conditional on compatibility. If she accepts, the first-ranked agent and the item leave the market. If the item is not compatible with the first-ranked agent, or if it is compatible but the agent rejects it, the item is offered to the second-ranked agent (if there is any), conditional on compatibility. If the match occurs, the second-ranked agent and the item leave the market. Otherwise, the item is discarded. In addition, at the end of each period, if an old agent is still unmatched, she leaves. Therefore, one item and either one or two agents leave the market at each period, either because they match or because their lifespan ends.

For simplicity, we assume that an agent indifferent between accepting and rejecting an item always accepts it. Our analysis remains unchanged under any other tie-breaking rule.

## 2.2 Rational and Behavioral Equilibrium

We study stationary Markov perfect equilibria, in which agents’ strategies only depend on their age (young or old) and their rank in the queue. Since an old agent has a weakly dominant strategy in accepting any item offered to her rather than leaving the market unmatched, an equilibrium is fully described by young agents’ behavior. While the rational benchmark requires the minimum acceptable item’s value to coincide with an agent’s continuation value, we consider behavioral equilibria, in which agents’ thresholds can differ from it.

### 2.2.1 First-In-First-Out

In the FIFO protocol, an old agent is always ranked first. Thus, a young agent’s continuation value, and therefore her minimum acceptable value, are independent of her ranking upon

arrival.

At the beginning of each period, there are either two agents (one old and one young, ranked first and second, respectively) or only one (young) agent on the market. Let  $\{v_o^F, v_{y2}^F, v_{y1}^F\}$  denote the expected payoffs of the agents in these scenarios at the beginning of a period, before the item's value is realized. Suppose that a young agent uses a threshold strategy  $\hat{\theta} \in [0, 1]$ . If there are two agents, one old and one young, their payoffs are:

$$\begin{aligned} v_o^F &= \frac{p}{2}, \\ v_{y2}^F &= p(1 - \hat{\theta}) \left[ (1 - p) \frac{1 + \hat{\theta}}{2} + pv_o^F \right] + [1 - p(1 - \hat{\theta})] v_o^F. \end{aligned} \quad (1)$$

The old agent is ranked first and accepts any compatible item, yielding  $\frac{p}{2}$ . Consider the young agent, ranked second. The item is both compatible with her and acceptable with probability  $p(1 - \hat{\theta})$ . In this scenario, the young agent obtains the item with expected value  $\frac{1 + \hat{\theta}}{2}$  only if it is incompatible with the old agent (i.e., with probability  $1 - p$ ). Otherwise, the young agent remains unmatched with continuation value  $v_o^F$ . If no old agent is present, the young agent's expected payoff is:

$$v_{y1}^F = p(1 - \hat{\theta}) \frac{1 + \hat{\theta}}{2} + [1 - p(1 - \hat{\theta})] v_o^F. \quad (2)$$

We denote the rational Markov perfect equilibrium threshold and expected payoffs under FIFO by  $(\bar{\theta}^F, \bar{v}_o^F, \bar{v}_{y2}^F, \bar{v}_{y1}^F)$ , where  $\bar{\theta}^F = \bar{v}_o^F = \frac{p}{2}$ .

Next, we consider a behavioral equilibrium where a young agent's decision can differ from the rational choice. Namely, we have

$$\tilde{\theta}^F = \alpha v_o^F = \alpha \frac{p}{2}, \quad (3)$$

where  $0 < \alpha < \frac{2}{p}$  to guarantee  $0 < \tilde{\theta}^F < 1$ . While  $\alpha = 1$  corresponds to the rational benchmark, we refer to agents with  $\alpha > 1$  and  $\alpha < 1$  as *overselective* and *undersselective* under FIFO, respectively. For any  $\alpha \in (0, \frac{2}{p})$ , we denote the unique solution of the equilibrium conditions (1), (2), and (3) by  $(\tilde{\theta}^F, \tilde{v}_o^F, \tilde{v}_{y2}^F, \tilde{v}_{y1}^F)$ .

### 2.2.2 Last-In-First-Out

In the LIFO protocol, a young agent is always ranked first. If all young agents play a threshold strategy  $\hat{\theta} \in [0, 1]$ , an old (second-ranked) agent's expected payoff at the beginning of the period is

$$v_o^L = (1 - p)\frac{p}{2} + p\hat{\theta}\frac{p\hat{\theta}}{2}. \quad (4)$$

The old agent can obtain an item if the item is compatible with her (probability  $p$ ) and either incompatible with the young agent (probability  $1 - p$  and expected value  $\frac{1}{2}$ ) or compatible but unacceptable (probability  $p\hat{\theta}$  and expected value  $\frac{\hat{\theta}}{2}$ ).

A young agent's expected payoff is

$$v_y^L = p(1 - \hat{\theta})\frac{1 + \hat{\theta}}{2} + [1 - p(1 - \hat{\theta})]v_o^L. \quad (5)$$

Since the young agent is ranked first, she obtains the item if it is compatible and acceptable to her (probability  $p(1 - \hat{\theta})$ ), yielding the expected payoff  $\frac{1 + \hat{\theta}}{2}$ . Otherwise, her continuation value is  $v_o^L$ .

Letting  $(\bar{\theta}^L, \bar{v}_y^L, \bar{v}_o^L)$  be the rational Markov perfect equilibrium threshold and payoffs under LIFO, it is easy to check that  $\bar{\theta}^L = \bar{v}_o^L = \frac{1 - \sqrt{1 - p^3(1 - p)}}{p^2}$ .

Again, a behavioral equilibrium allows agents to use thresholds different from their continuation values. Namely, we consider

$$\tilde{\theta}^L = \beta v_o^L \quad (6)$$

for some  $\beta > 0$ . While  $\beta = 1$  corresponds to the rational benchmark, we refer to agents with  $\beta > 1$  and  $\beta < 1$  as *overselective* and *underselective* under LIFO, respectively. We denote the unique solution of the equilibrium conditions (4), (5), and (6) by  $(\tilde{\theta}^L, \tilde{v}_y^L, \tilde{v}_o^L)$ . In particular, we obtain

$$\tilde{\theta}^L = \frac{1 - \sqrt{1 - \beta^2 p^3(1 - p)}}{\beta p^2}. \quad (7)$$

Again,  $0 < \beta < \frac{2}{p}$  guarantees  $0 < \tilde{\theta}^L < 1$ .

The equilibrium threshold  $\tilde{\theta}^L$  increases in  $\beta$ . Naturally, as  $\beta$  increases, for the same continuation value  $v_o^L$ , an agent becomes more selective. In addition, because each agent's continuation value  $v_o^L$  increases as other agents become more selective, there is an indirect effect driven by strategic complementarity. More formally, if an agent is overselective ( $\beta > 1$ ),

but others are not and play the rational equilibrium strategy  $\bar{\theta}^L$ , then the agent's threshold would be  $\beta\bar{v}_o^L < \tilde{\theta}^L$ . The opposite applies if an agent is underselective ( $\beta < 1$ ) but the others are not. The following lemma illustrates this effect.

**Lemma 1.** *Under LIFO, the behavioral equilibrium threshold  $\tilde{\theta}^L$  is such that  $\tilde{\theta}^L > \beta\bar{v}_o^L$  for  $\beta > 1$ , and  $\tilde{\theta}^L < \beta\bar{v}_o^L$  for  $\beta < 1$ .*

## 2.3 Equilibrium and Efficiency Comparisons

Next, we compare the rational equilibria under FIFO and LIFO, and study how behavioral patterns influence this comparison. We start with some preliminaries. For  $h = F, L$ , given a threshold  $\hat{\theta} \in (0, 1)$  used by all agents, we denote by  $W^h(\hat{\theta})$  the expected time that an agent stays on the market, which coincides with the probability that an agent is not matched upon arrival. Since the probability of matching on arrival under LIFO is  $1 - W^L(\hat{\theta}) = p(1 - \hat{\theta})$ , we have

$$W^L(\hat{\theta}) = 1 - p(1 - \hat{\theta}). \quad (8)$$

Under FIFO,  $W^F(\hat{\theta})$  has to satisfy

$$W^F(\hat{\theta}) = (1 - W^F(\hat{\theta}))[1 - p(1 - \hat{\theta})] + W^F(\hat{\theta})[1 - (1 - p)p(1 - \hat{\theta})]. \quad (9)$$

This is because a young agent is ranked first on arrival if and only if her immediate predecessor is matched on arrival (with probability  $1 - W^h(\hat{\theta})$ ). Then, that agent stays on the market into the second period with probabilities  $1 - p(1 - \hat{\theta})$  and  $1 - (1 - p)p(1 - \hat{\theta})$  if she is ranked and second on arrival, respectively. Solving (9) for  $W^F(\hat{\theta})$ , we obtain

$$W^F(\hat{\theta}) = 1 - \frac{(1 - p)p(1 - \hat{\theta})}{1 - p^2(1 - \hat{\theta})}. \quad (10)$$

We measure the efficiency of a queuing protocol by its average match value. More formally, given a threshold  $\hat{\theta} \in (0, 1)$  used by all agents, the *allocation efficiency* of protocol  $h = F, L$  is given by

$$E^h(\hat{\theta}) = \frac{pW^h(\hat{\theta})}{2} + (1 - pW^h(\hat{\theta}))\frac{p(1 - \hat{\theta}^2)}{2}, \quad (11)$$

where  $W^h(\hat{\theta})$  is determined by (8) for LIFO and by (10) for FIFO.<sup>4</sup> The first term of (11)

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<sup>4</sup>Alternatively, the same allocation efficiency (11) can be derived from the agents' average payoffs: given



represents the scenario in which an old agent is present on the market and is compatible with an item (with probability  $pW^h(\hat{\theta})$ ). Then, the item is matched for sure, yielding an expected efficiency gain of  $\frac{1}{2}$ . Otherwise, only a young agent can obtain the item. This happens if the item is both compatible and acceptable to her. Therefore, with probability  $p(1 - \hat{\theta})$ , the expected efficiency gain is  $\frac{1+\hat{\theta}}{2}$ , resulting in the second term of (11).

### 2.3.1 Allocation Efficiency in the Rational Equilibrium

Setting  $\alpha = \beta = 1$ , we now compare the efficiency in the rational equilibrium with the socially optimal (i.e., efficiency maximizing) stationary mechanisms under FIFO and LIFO.<sup>5</sup> Our first result describes an optimal mechanism.<sup>6</sup>

**Proposition 1.** *1. A socially optimal stationary allocation mechanism is a FIFO protocol with*

$$\theta^{F-opt} = \frac{-(1 - p^2) + \sqrt{1 - 2p^2 + p^3}}{p^2}; \quad (12)$$

*2. We have  $0 < \theta^{F-opt} < \bar{\theta}^F$ .*

Subject to compatibility, a social planner prefers to match an item with an old agent rather than a young agent. This is because, while the surplus of these two matches is the same, the young agent could realize some positive surplus by staying on the market through the next period. Since the planner prioritizes old agents (i.e., uses FIFO), an optimal mechanism is identified by the threshold  $\theta^{F-opt}$  that maximizes the efficiency (11) under FIFO, resulting in Part (1) of Proposition 1.

For Part (2) of Proposition 1, suppose a young agent under FIFO is offered an item of value  $\bar{\theta}^F$ , and is therefore indifferent between accepting or rejecting it. Rejection strictly increases the probability of the next agent remaining unmatched, since, conditional on compatibility with both agents, the next item will not be offered to her. Therefore, a social planner strictly prefers that match to occur, implying the rational threshold  $\bar{\theta}^F$  being strictly above the optimal one  $\theta^{F-opt}$ —that is, rational agents under FIFO are too selective compared to the social optimum.

While Proposition 1 guarantees that in this setting, a social planner prefers a FIFO protocol to a LIFO one, the next result identifies the most efficient threshold under LIFO.

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$\hat{\theta} \in (0, 1)$ , we have  $E^L(\hat{\theta}) = v_y^L$  under LIFO, and  $E^F(\hat{\theta}) = (1 - W^F(\hat{\theta}))v_{y1}^F + W^F(\hat{\theta})v_{y2}^F$  under FIFO.

<sup>5</sup>A stationary mechanism determines an item's allocation only based on the item's value and the presence of an old agent on the market.

<sup>6</sup>The proof of Proposition 1 follows easily from (10) and (11), and is therefore omitted.

**Proposition 2.** *The optimal threshold  $\theta^{L-opt}$  under LIFO is such that  $\theta^{L-opt} > \bar{\theta}^L$ .*

To understand Proposition 2, suppose a young agent is offered an item of value  $\bar{\theta}^L$  under LIFO, and is therefore indifferent between accepting or rejecting it. If an old agent is present and the item is compatible with her, then by allocating the item to the old agent, she leaves the market matched rather than unmatched, generating a strictly positive surplus. Moreover, the continued presence of the young agent in the next period generates no externalities on future agents, as the agent will be ranked second and, therefore, may obtain the next item only when it would be wasted otherwise. Thus, a social planner strictly prefers the young agent to reject, implying that the equilibrium threshold  $\bar{\theta}^L$  is below the optimal one  $\theta^{L-opt}$ , i.e. rational agents under LIFO are too accommodating compared to the social optimum.

The next result compares the rational equilibrium under FIFO and LIFO, and their respective allocation efficiencies.

**Proposition 3.** *In the rational equilibrium,*

1. *Agents are more selective under FIFO than under LIFO—that is,  $\bar{\theta}^L < \bar{\theta}^F$ ;*
2. *The expected time on the market under FIFO is longer than under LIFO—that is,  $W^F(\bar{\theta}^F) > W^L(\bar{\theta}^L)$ ;*
3. *The allocation efficiency is higher under FIFO than under LIFO—that is,  $E^F(\bar{\theta}^F) > E^L(\bar{\theta}^L)$ .*

Part (1) of Proposition 3 holds because young agents' positions deteriorate more under LIFO than under FIFO, making them relatively less selective. Therefore, on average, fewer old agents are present on the market under LIFO, as highlighted in Part (2). Part (3) of Proposition 3 shows that, in our setting, the FIFO rational equilibrium is more efficient than the LIFO one.

To gain intuition about Part (3), we compare the expected payoffs of a young agent arriving as ranked first under FIFO, as ranked second under FIFO, and under LIFO (where she is always first), respectively, conditional on *all agents using the same strategy*  $\hat{\theta} \in (0, 1)$ . The expected payoff of a young agent who is ranked first under FIFO is  $v_{y1}^F$ , as described in (2). Suppose now that the young agent is ranked second in FIFO rather than first. The lower rank affects the agent's payoff only if the arriving item is compatible with and acceptable to the agent, and also compatible with the (first-ranked) old agent. This event occurs with

probability  $p^2(1 - \hat{\theta})$ , and in this case, the young agent receives a continuation value of  $\frac{p}{2}$ , rather than  $\frac{1+\hat{\theta}}{2}$ , implying

$$v_{y1}^F - v_{y2}^F = p^2(1 - \hat{\theta}) \left( \frac{1 + \hat{\theta}}{2} - \frac{p}{2} \right).$$

Finally, consider a young agent under LIFO, rather than ranked first under FIFO. The change in protocol affects the agent's payoff only if the arriving item is either incompatible or unacceptable, which occurs with probability  $1 - p(1 - \hat{\theta})$ . Conditional on this scenario, her payoff changes only if next period's item is compatible with both her and the next agent, which occurs with probability  $p^2$ . Then, our agent, who is now old, surely obtains the next item under FIFO (with expected value  $\frac{1}{2}$ ), while under LIFO she obtains it only if the item's value is unacceptable to the next agent (probability  $\hat{\theta}$ ), with expected value  $\frac{\hat{\theta}}{2}$ . Hence,

$$v_{y1}^F - v_y^L = \left[ 1 - p(1 - \hat{\theta}) \right] p^2 \left( \frac{1}{2} - \hat{\theta} \frac{\hat{\theta}}{2} \right).$$

For any  $\hat{\theta} \in (0, 1)$ , it is easy to verify that  $v_{y1}^F > v_{y2}^F > v_y^L$ —that is, being second upon arrival under FIFO is preferable to entering a market under LIFO. In particular, this holds for  $\hat{\theta} = \bar{\theta}^L$ . Furthermore, if an agent under FIFO chooses  $\bar{\theta}^F = \frac{p}{2}$  instead of  $\bar{\theta}^L$ , her payoff is even higher. Thus, the expected payoff of an arriving agent under FIFO, which is a weighted average of the values of being ranked first or second upon arrival, must be higher than under LIFO, yielding Part (3) of Proposition 3.

### 2.3.2 Allocation Efficiency in the Behavioral Equilibrium

Next, we describe how behavioral patterns affect the previous results. We begin by formalizing a minimal amount of rationality in queues.

**Definition 1.** *An agent is “minimally queuing-rational” (MQ-rational) if her strategies  $\theta^L, \theta^F$  are such that (i)  $\theta^L, \theta^F \leq 1/2$ , and (ii)  $\theta^L \leq \theta^F$ .<sup>7</sup>*

Requirement (i) of MQ-rationality guarantees that an agent always accepts any item of value greater than  $1/2$ , which is the continuation value if the next period's item is both compatible and allocated to the agent for sure. Requirement (ii) amounts to an agent

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<sup>7</sup>MQ-rationality translates into bounds for  $\alpha$  and  $\beta$ . Specifically,  $\alpha \leq \frac{1}{p}$  and, if all agents are MQ-rational, then  $\beta \leq \bar{\beta}$ , where  $\bar{\beta}$  is the unique positive solution of  $\frac{1 - \sqrt{1 - \beta^2 p^3 (1 - p)}}{\beta p^2} = \frac{\alpha p}{2}$ .

understanding that her future position deteriorates more under LIFO than under FIFO. It is easy to verify that Parts (1) and (2) of Proposition 3 continue to hold in behavioral equilibrium if agents are (strictly) MQ-rational.

However, if MQ-rational agents under LIFO become increasingly selective – that is, if  $\tilde{\theta}^L$  moves toward  $\tilde{\theta}^F$  or, equivalently,  $\beta$  increases – the difference in expected wait time between LIFO and FIFO decreases. Proposition 4 illustrates how behavioral patterns under LIFO influence the equilibrium efficiency and how it compares with the FIFO protocol.

**Proposition 4.** *If agents are MQ-rational,*

1. *The allocation efficiency of the LIFO equilibrium is a single-peaked function of  $\beta$ , and is maximal at some  $\beta^* > 1$ ;*
2. *We have  $E^L(\theta^{L-opt}) < E^F(\bar{\theta}^F)$ . Moreover, the efficiency gap between the FIFO equilibrium at  $\alpha = 1$ , and the LIFO equilibrium decreases for  $\beta \in [1, \beta^*]$  and increases for  $\beta \in (\beta^*, \bar{\beta}]$ .*

Part (1) of Proposition 4 follows from Proposition 2: Efficiency is maximized at a threshold higher than  $\bar{\theta}^L$ , associated with  $\beta^* > 1$ . Part (2) guarantees that the efficiency under LIFO, even at the optimal threshold, is lower than the FIFO rational equilibrium efficiency. The intuition is almost identical to Part (3) of Proposition 3, substituting  $\bar{\theta}^L$  with  $\theta^{L-opt}$ .

Proposition 4 summarizes the efficiency implication of agents departing from rational behavior under LIFO. In particular, if the agents are moderately overselective under LIFO ( $1 < \beta < \beta^*$ ), such departure is efficiency-improving, and it reduces the efficiency gap between LIFO and FIFO. However, if they become extremely overselective ( $\beta > \beta^*$ ), the efficiency gap starts to grow again and can ultimately surpass the one associated with the rational equilibrium. Proposition 4 constitutes the underpinning of the efficiency implications of our experimental analysis, which we present next.

## 3 Experimental Design

### 3.1 Reduced-Form Games

To replicate an infinite-horizon overlapping generation game in an experimental setting, we follow the approach of Lim et al. (1986), Aliprantis and Plott (1992), and Marimon and Sunder (1993). Specifically, we consider reduced-form static setups, which are strategically equivalent to the FIFO and LIFO environments presented in Section 2.

**Reduced-Form FIFO** A single player selects a threshold  $\tilde{\theta} \in (0, 1)$ . Then, an item with value  $\theta_1 \sim U[0, 1]$  arrives and is compatible with the player with probability  $p \in (0, 1)$ . If the item is compatible and  $\theta_1 \geq \tilde{\theta}$ , the player receives the payoff  $\theta_1$ , and the process ends. Otherwise, a second item arrives with value  $\theta_2 \sim U[0, 1]$  and compatible with probability  $p \in (0, 1)$ . If the second item is compatible, the player’s payoff is  $\theta_2$ . Otherwise, the player obtains zero.

This decision problem is equivalent to a FIFO protocol described in Section 2.2.1, and the strategy chosen by a player with a behavioral parameter  $\alpha \in (0, \frac{2}{p})$  is  $\tilde{\theta}^F = \alpha \frac{p}{2}$ .

**Reduced-Form LIFO** Players A and B simultaneously select their thresholds  $\tilde{\theta}_A \in (0, 1)$  and  $\tilde{\theta}_B \in (0, 1)$ , respectively. Then, either player is equally likely to be chosen as ‘active’ player. The other player obtains a payoff of zero.

Suppose that Player  $i = A, B$  is chosen to be active. An item with value  $\theta_1 \sim U[0, 1]$  arrives and is compatible with Player  $i$  with probability  $p \in (0, 1)$ . If the item is compatible with Player  $i$  and  $\theta_1 \geq \tilde{\theta}_i$ , then Player  $i$  obtains  $\theta_1$  and the game ends. Otherwise, a second item arrives with value  $\theta_2 \sim U[0, 1]$ , compatible with each player with probability  $p \in (0, 1)$ , with compatibility independent between players. Then, one of these cases follows:

1. If  $\theta_2 \geq \tilde{\theta}_{-i}$  and the second item is not compatible with Player  $-i$  but is compatible with Player  $i$  (with probability  $p(1 - p)$ ), Player  $i$  obtains  $\theta_2$ .
2. If  $\theta_2 < \tilde{\theta}_{-i}$  and the second item is compatible with Player  $i$  (with probability  $p$ ), then Player  $i$  obtains  $\theta_2$ .
3. Otherwise, Player  $i$  obtains zero.

This setup is strategically equivalent to the LIFO protocol studied in Section 2.2.2. If the players of this game have a behavioral parameter  $\beta \in (0, \frac{2}{p})$ , there exists a unique behavioral equilibrium where both choose  $\tilde{\theta}^L$ , described in (7).

## 3.2 Experiment Description

The experiment took place at Washington University in Saint Louis, with 60 undergraduate students as participants. Each session comprises 30 rounds of the FIFO game and 30 rounds of the LIFO game. We briefly describe the experiment; detailed instructions are in the Supplemental Appendix.

Subjects are randomly paired at the start of each round in both FIFO and LIFO.<sup>8</sup> Items are represented by jars of tokens. Each subject selects an integer between 1 and 1000 as minimum acceptable jar value. Then, one subject in each pair is randomly chosen as ‘active’.

In the *FIFO treatment*, a computer generates the first jar with a random value. If the value meets or exceeds the active subject’s threshold, that subject obtains the jar with probability 50%, concluding the round. Otherwise, a second jar is generated. Regardless of its value, the active subject has a 50% chance of obtaining the second jar. If she does not obtain it, she receives a payoff of zero, and the round ends.

In the *LIFO treatment*, a computer generates the first jar with a random value. If the value meets or exceeds the active subject’s threshold, the active subject obtains the jar with a 50% chance, concluding the round. Otherwise, a second jar is generated. If the second jar’s value meets or exceeds the inactive subject’s threshold, the active subject has a 25% chance of obtaining the jar. If the second jar’s value is strictly below the inactive subject’s threshold, the active subject has a 50% chance of obtaining it. If the active subject fails to obtain the second jar, she receives zero, ending the round.

## 4 Experimental Results

### 4.1 Main Analysis

First, we assess whether the observed choices satisfy MQ-rationality, formalized in Definition 1. MQ-rationality requires a subject’s threshold choices under LIFO to be lower than under FIFO, and both to be at most 500.

**Result 1.** *Subjects are MQ-rational, i.e., their choices in both treatments are mostly lower than 500, and lower under LIFO than under FIFO.*

Exhibit 1 describes the threshold choices of each treatment. Since the subjects may gradually learn, we present three data sets: all 30, last 15, and last 5 rounds.

The last three columns of the table in Exhibit 1 summarize the experimental data. The second, third, and fourth column of the table show the mean and standard deviation (in parentheses) of the subjects’ choices across all 30, last 15, and last 5 rounds, respectively, which are all consistent with MQ-rationality.

We also observe MQ-rationality at individual levels. The figures in Exhibit 1 show each subject’s average choices (each dot representing one subject), averaged across all 30, last 15,

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<sup>8</sup>The pairing is strategically irrelevant under FIFO but ensures a similar structure between treatments.

Equilibrium Threshold		Experimental Data		
		All rounds	Last 15	Last 5
FIFO	250	270.6 (23.1)	258.5 (26.8)	266.8 (28.2)
LIFO	127	218.3 (22.9)	197.8 (24.7)	193.5 (27.1)

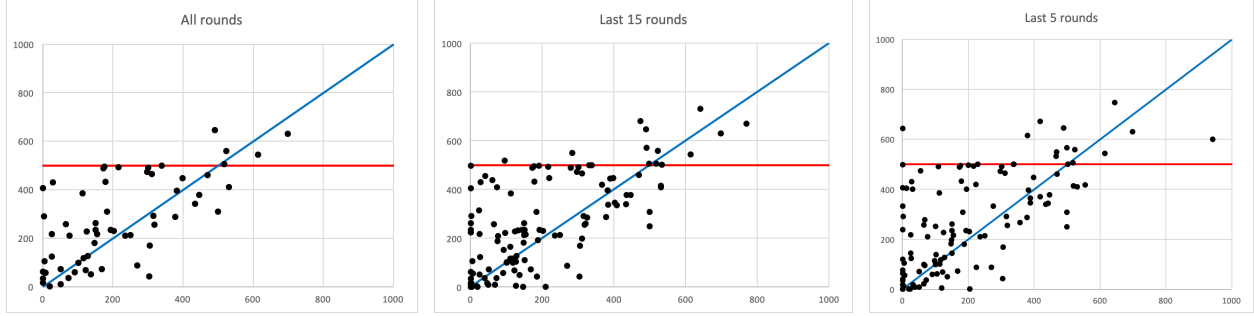


Exhibit 1: Subjects' Threshold Choices. The table shows the choices averaged across all subjects. The figures show individual subjects' average choices under LIFO (on the  $x$ -axis) and FIFO (on the  $y$ -axis).

or last 5 rounds under LIFO (on the  $x$ -axis) and FIFO (on the  $y$ -axis). Hence, the dots in the bottom-left triangle of each plot represent subjects that satisfy MQ-rationality on average.

The subjects' threshold choices either satisfy MQ-rationality or do not deviate enough from it to reject the MQ-rationality hypothesis. Let  $\{\hat{\theta}_i^F, \hat{\theta}_i^L\}_{i=1}^{60}$  be the observed threshold choices under FIFO and LIFO. Assuming that the chosen thresholds are independent samples of the equilibrium thresholds  $(\tilde{\theta}^F, \tilde{\theta}^L)$ , the 95% confidence interval for the FIFO threshold  $\tilde{\theta}^F$  is  $[224.4, 316.8]$ ,  $[204.8, 312.2]$ , or  $[210.4, 323.1]$  for the data from all 30, last 15, and last 5 rounds, respectively. Similarly, the 95% confidence interval for the threshold difference,  $\tilde{\theta}^F - \tilde{\theta}^L$ , is  $[15.3, 89.3]$ ,  $[19.2, 102.2]$ , or  $[26.2, 120.3]$  for the data from all 30, last 15, and last 5 rounds. One-sided tests reject the hypotheses  $\tilde{\theta}^F \geq 500$  and  $\tilde{\theta}^L \geq \tilde{\theta}^F$ .

Next, we compare the subjects' choices to the rational equilibria. As shown in Exhibit 1, rationality ( $\alpha = \beta = 1$ ) entails the equilibrium threshold under FIFO and LIFO of  $\bar{\theta}^F = 250$  and  $\bar{\theta}^L = 127$ , respectively.

**Result 2.** (i) *The subjects' behavior is close to rational in FIFO, but overselective in LIFO;* (ii) *The observed difference between the FIFO and LIFO thresholds is smaller than in the rational equilibrium.*

Exhibit 2 illustrates the evolution of the subjects' choices over all rounds. For each round

of each treatment, the figure shows the mean of the subjects' choices (in solid lines) and the 95% confidence interval (in the shaded areas).

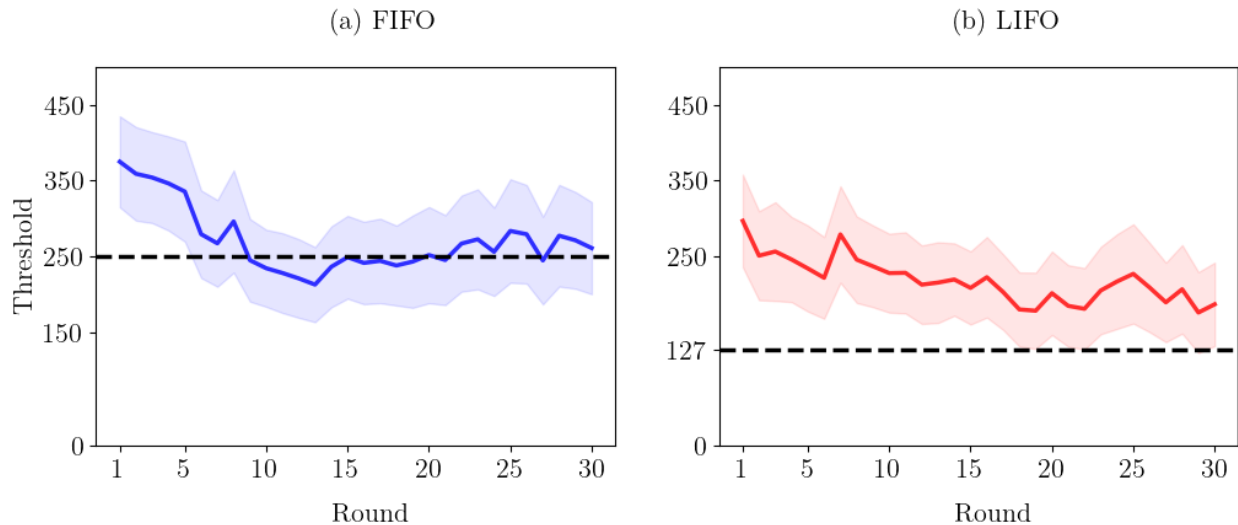


Exhibit 2: Rational vs. Observed Choices under FIFO [Panel (a)] and LIFO [Panel (b)]

Panel (a) depicts the choices under FIFO, and the rational threshold of 250. The threshold choices are generally aligned with the rational benchmark, starting higher in the early rounds but converging to 250 within 5-10 rounds. Panel (b) shows the choices under LIFO, and the rational equilibrium threshold of 127. Although the mean of the chosen thresholds tend to decrease slightly over time, they remain significantly higher than the rational equilibrium of 127. Therefore, from Results 1 and 2, we conclude that the observed difference between the FIFO and LIFO thresholds is positive, but smaller than what the rational equilibrium predicts.

Lastly, we estimate the behavioral parameters  $\alpha$  and  $\beta$  using the observed choices.

**Result 3.** (i) *The estimated behavioral parameters from the threshold choices from all, last 15, and last 5 rounds, are  $\hat{\alpha} \in [1.03, 1.08]$  and  $\hat{\beta} \in [1.49, 1.67]$ .* (ii) *Such estimates imply a reduction in the allocation efficiency gap between the FIFO and LIFO protocols compared to the rational equilibrium.*

The estimate of the behavioral parameter  $\hat{\alpha}$  is derived from  $\hat{\theta}^F = \hat{\alpha}^p$ , with  $p = \frac{1}{2}$ . Here,  $\hat{\theta}^F$  represents the threshold choices averaged across all, the last 15, or the last 5 rounds, resulting in values of 270.6, 258.5, or 266.8, respectively, as shown in Exhibit 1. Similarly, the estimate of the parameter  $\hat{\beta}$  is obtained from (7) and Exhibit 1.



The behavioral patterns described in Result 3 allow to draw the efficiency implications of our experimental results. According to Part (2) of Proposition 4, the overselective behavior under LIFO has the potential to enhance efficiency, if it is not too severe. With  $p = 0.5$ , we have  $\bar{\theta}^L = 0.127$  and  $\theta^{L-opt} = 0.184$ , corresponding to  $\beta^{L-opt} = 1.42$ .

Hence, the observed thresholds under LIFO in Exhibit 1 are excessively high, leading to a decrease in the efficiency relative to the maximal one. However, even the highest mean threshold choice of  $\hat{\theta}^L = 0.218$  from all 30 rounds satisfies  $E^L(\hat{\theta}^L) > E^L(\bar{\theta}^L)$ . The allocation efficiency under the observed behavior is lower than the optimal level but still higher than what rationality would entail. Therefore, the behavioral pattern we document *reduces the efficiency gap between the FIFO and LIFO protocols with respect to the rational equilibrium and helps mitigate the efficiency loss caused by the LIFO protocol.*

Our experimental results have broader implications, including setups where the welfare ranking between FIFO and LIFO is reversed. In these cases, the welfare consequences of the overselective bias under LIFO need to be evaluated carefully, depending on the model at hand. For example, consider a setting similar to ours, where agents must pay a positive waiting cost if they enter their old age still unmatched. In such a setting, a LIFO protocol generates an efficiency benefit relative to FIFO, since, as in Part (2) of Proposition 3, equilibrium waits are going to be longer in FIFO. If waiting costs are high enough, Part (3) of Proposition 3 reverses, and LIFO becomes more efficient than FIFO under rational behavior. Note, however, that, since Proposition 2 would still hold, a moderate overselective bias under LIFO would still entail an efficiency gain. Then, our results suggest that both the performance of LIFO and the efficiency gap between the two protocols increase with respect to the rational benchmark.

Finally, the observed behavior conflicts with standard economic theories. Time discounting, waiting costs, and risk aversion would all imply  $\alpha < 1$  and  $\beta < 1$ . Moreover, subjects' continuation values under LIFO are more complex to compute than under FIFO, as they depend on beliefs about another agent's choice. Therefore, aversion to complexity or ambiguity would lead to a more cautious behavior under LIFO than FIFO, implying  $\beta < \alpha$ , which is the opposite of what we find. In the next section, we address the impact of strategic considerations in our results.

## 4.2 Human-to-Robot LIFO Treatment

While choices under FIFO are non-strategic, a subject's continuation value under LIFO depends on the opponent's threshold choice, as illustrated in Lemma 1. If subjects form

beliefs about their opponents' choices based on observed high thresholds from earlier rounds, they may not adjust their choices downward, thereby preventing convergence to the rational equilibrium. To address this effect, we conducted an additional LIFO treatment, where 47 subjects played the reduced-form LIFO game against robots with known strategies.

In this additional treatment, a subject selects an integer between 1 and 1000 for each of the 7 possible robot choices  $Z \in \{1, 100, 200, 300, 400, 500, 1000\}$ . For each robot's choice a subject's response, we apply the LIFO treatment described in Section 3.2. The rational best response to a robot's choice can be derived by replacing  $\hat{\theta}$  with  $z := \frac{Z}{1000} \in [0, 1]$  in (4) to obtain  $\bar{\theta}(z) = \frac{1}{2}[p(1-p) + p^2z^2]$ , and multiplying by 1000.

The human-to-robot LIFO treatment was repeated for 5 rounds. Exhibit 3 shows that the subjects' average choices still consistently exceeded the rational best responses to the robot's choices.

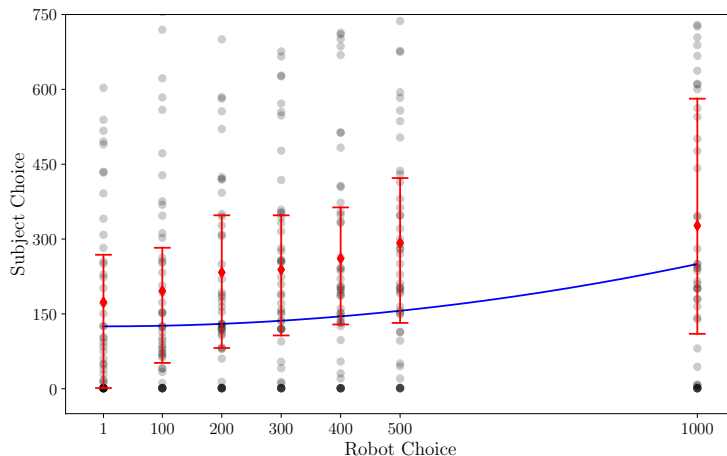


Exhibit 3: Subjects' Choices in the Human-to-Robot LIFO Treatment. A gray dot represents a subject's average threshold choice across 5 rounds. Red diamonds and segments represent observed averages and 25-to-75 percentile ranges, respectively. The blue curve is the rational best response.

We estimate a subject's behavioral parameter  $\hat{\beta}$  using (6), as the ratio of their threshold choice  $\hat{\theta}$  to the rational best response  $\bar{\theta}(z)$  for each robot's choice  $z$ . After averaging this estimation across all rounds and subjects, the resulting estimate  $\hat{\beta}$  falls within the range of [1.31, 1.56], which is close to the range [1.49, 1.67] in Result 3 of the original LIFO treatment. To summarize, *in the human-to-robot LIFO treatment, without strategic complementarities, subjects still exhibit an overselective bias similar to the one found in the original LIFO treatment.* In the Supplemental Appendix, we explore to what extent some well-known

behavioral frameworks can explain this residual bias, but further research could be beneficial to understand the causes of this departure from rationality.

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## 5 Appendix

**Proof of Lemma 1** When  $\beta = 1$ , we have  $\bar{\theta}^L = \bar{v}_o^L$ . For  $\beta < 1$ , let  $x = \beta^2$  and  $y = p^3(1-p)$ . Then,  $\tilde{\theta}^L < \beta\bar{v}_o^L = \beta\bar{\theta}^L$  if and only if

$$\frac{1 - \sqrt{1 - xy}}{x} < 1 - \sqrt{1 - y}.$$

To verify the last inequality, observe that the two sides of it are equal at  $x = 1$  and

$$\begin{aligned} \frac{d\left(\frac{1 - \sqrt{1 - xy}}{x}\right)}{dx} > 0 &\iff \frac{xy}{2}(1 - xy)^{-1/2} > 1 - \sqrt{1 - xy} \\ &\iff 1 - \frac{xy}{2} - \sqrt{1 - xy} > 0 \\ &\iff 0 < xy < 1, \end{aligned}$$

which always holds. A similar argument applies if  $\beta > 1$ . ■

**Proof of Proposition 2** From (8) and (11), we obtain

$$\theta^{L-opt} = \frac{1 - p + p^2 - \sqrt{(1 - p + p^2)^2 - 3p^3(1 - p)}}{3p^2}. \quad (13)$$

Recall that  $\bar{\theta}^L = \frac{1 - \sqrt{1 - p^3(1-p)}}{p^2}$ . Let  $x = p^2$  and  $y = p(1-p)$ . Then,  $\bar{\theta}^L < \theta^{L-opt}$  if and only if

$$\begin{aligned} 1 - \sqrt{1 - xy} &< \frac{(1 - y) - \sqrt{(1 - y)^2 - 3xy}}{3} \\ \iff (2 + y) &< 3\sqrt{1 - xy} - \sqrt{(1 - y)^2 - 3xy} \\ \iff (1 - y)(3 + y) &< 3 + xy, \end{aligned}$$

which always holds since  $(1 - y)(3 + y) = 3 - 2y - y^2 < 3$ . ■

**Proof of Proposition 3 Part (1)** For  $h = F, L$ , we have  $\bar{\theta}^h = \bar{v}_o^h$ . Since  $\bar{v}_o^F = \frac{p}{2}$ , and  $\bar{v}_o^L$  is given by (4), it is easy to check that  $\bar{v}_o^F > \bar{v}_o^L$  for any  $\hat{\theta} \in (0, 1)$ .

**Part (2)** By comparing (8) and (10), observe that  $W^L(\hat{\theta}) < W^F(\hat{\theta})$  for any  $\hat{\theta} \in (0, 1)$ . Moreover,  $W^L(\hat{\theta}) = 1 - p(1 - \hat{\theta})$  strictly increases in  $\hat{\theta}$ . Thus, (2) follows from (1), as  $\bar{\theta}^L < \bar{\theta}^F$ .

**Part (3)** First, we show that, for any  $\hat{\theta} \in (0, 1)$ ,  $v_{y_2}^F(\hat{\theta}) > v_y^L(\hat{\theta})$ . From (1), we have

$$v_{y_2}^F(\hat{\theta}) = p(1 - \hat{\theta}) \left[ (1 - p) \frac{1 + \hat{\theta}}{2} + p \frac{p}{2} \right] + [1 - p(1 - \hat{\theta})] \frac{p}{2}.$$

From (4) and (5),

$$v_y^L(\hat{\theta}) = p(1 - \hat{\theta}) \frac{1 + \hat{\theta}}{2} + [1 - p(1 - \hat{\theta})] \left[ (1 - p) \frac{p}{2} + (p\hat{\theta}) \frac{p\hat{\theta}}{2} \right].$$

Then,

$$\begin{aligned} v_{y_2}^F(\hat{\theta}) - v_y^L(\hat{\theta}) &= p(1 - \hat{\theta}) \left[ \frac{p^2 - p(1 + \hat{\theta})}{2} \right] + [1 - p(1 - \hat{\theta})] \frac{p^2(1 - \hat{\theta}^2)}{2} \\ &= p(1 - \hat{\theta}) \frac{p^2 - p^2(1 - \hat{\theta}^2)}{2} > 0. \end{aligned} \tag{14}$$

Finally,

$$E^L(\bar{\theta}^L) = v_y^L(\bar{\theta}^L) < v_{y_2}^F(\bar{\theta}^L) < v_{y_2}^F(\bar{\theta}^F) < E^F(\bar{\theta}^F).$$

The first inequality follows from (14), and the subsequent inequalities hold because  $v_{y_2}^F(\hat{\theta})$  increases as  $\hat{\theta}$  approaches  $\bar{\theta}^F = \frac{p}{2}$ , and  $v_{y_2}^F(\bar{\theta}^F)$  must be lower than the average utility across all agents under FIFO. ■

**Proof of Proposition 4** Since  $E^L(\hat{\theta}) = v_y^L(\hat{\theta})$ ,  $E^L(\hat{\theta})$  can be derived from (8) and (11) as a cubic function of  $\hat{\theta}$ . Then,

$$\frac{dE^L(\hat{\theta})}{d\hat{\theta}} = \frac{p}{2} \underbrace{\left[ -2\hat{\theta} + p(1 - p + p\hat{\theta}^2) + (1 - p(1 - \hat{\theta}))2p\hat{\theta} \right]}_{:=h(\hat{\theta})}.$$

For Part (1), note that  $h(\hat{\theta})$  has a positive quadratic coefficient, that  $h(0) = p(1 - p) > 0$ , and that  $h\left(\frac{1}{2}\right) = p\left(2 - \frac{5p}{4}\right) - 1 < 0$ . Consequently,  $E^L(\hat{\theta})$  is a single-peaked function over the interval  $\hat{\theta} \in [0, \frac{1}{2}]$ , which we focus on by MQ-rationality.

Given that  $\tilde{\theta}^L$  strictly increases in  $\beta$  by (7),  $E^L(\hat{\theta})$  is a single-peaked function of  $\beta$ . Furthermore,  $\theta^{L-opt} > \bar{\theta}^L$  from Proposition 2, so the maximal efficiency is achieved at some  $\beta^* > 1$ .

For Part (2), observe that, since  $h(\frac{p}{2}) = p^3(\frac{3p}{4} - 1) < 0$ , we have  $\bar{\theta}^F = \frac{p}{2} > \theta^{L-opt}$ . Then,

$$E^L(\theta^{L-opt}) = v_y^L(\theta^{L-opt}) < v_{y2}^F(\theta^{L-opt}) < v_{y2}^F(\bar{\theta}^F) < E^F(\bar{\theta}^F).$$

The first inequality follows from (14), and the subsequent inequalities hold for reasons similar to the last part of the proof of Proposition 3. ■